

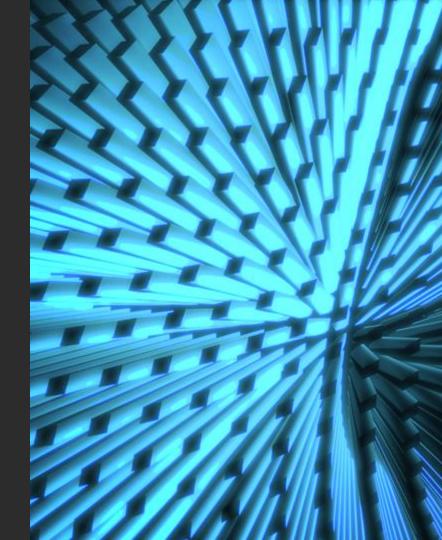
ApplicationDrivenLearning.jl A Closed-Loop Prediction and Optimization Approach

Joaquim Dias Garcia (Soma Energy) Giovanni Amorin (PUC-Rio, Brazil) Alexandre Street (PUC-Rio, Brazil)

Package: https://github.com/LAMPSPUC/ApplicationDrivenLearning.jl

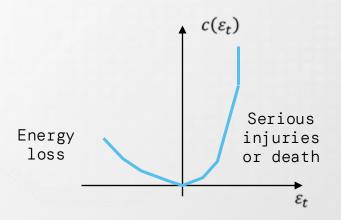
Theory paper: 10.1287/opre.2023.0565 (or https://arxiv.org/abs/2102.13273)

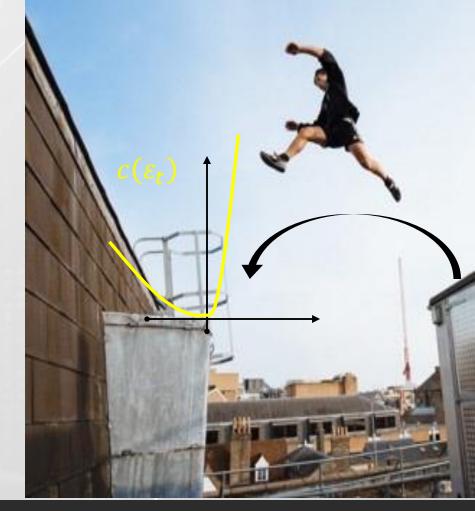
November 17th, 2025 - JuMP-dev 2025, Auckland, New Zealand



Motivation: Asymmetric costs

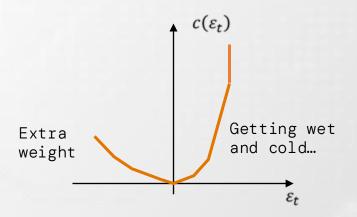
• Parkour: forecasted targets are biased





Motivation: Asymmetric costs

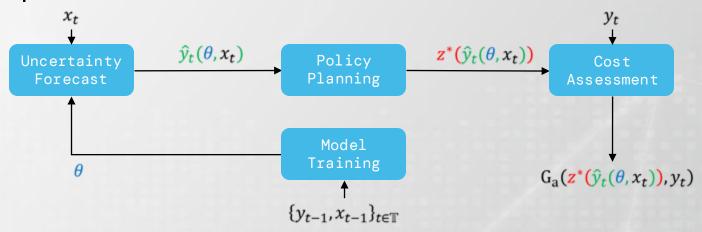
• Weather: forecasted targets are biased







Open-Loop



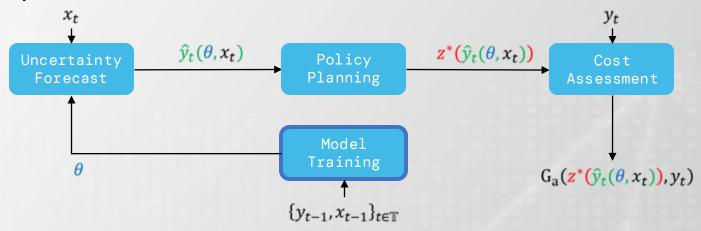
Train Load Forecast and Reserve Model

Forecast Loads and Reserve Requirements

Plan the Operation



Open-Loop



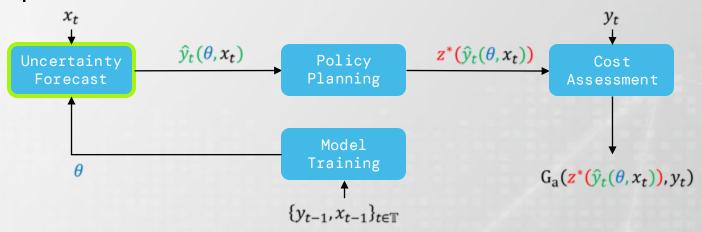
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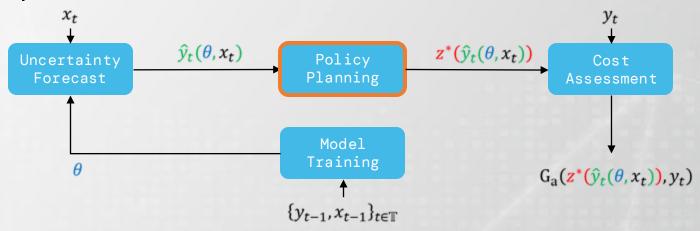
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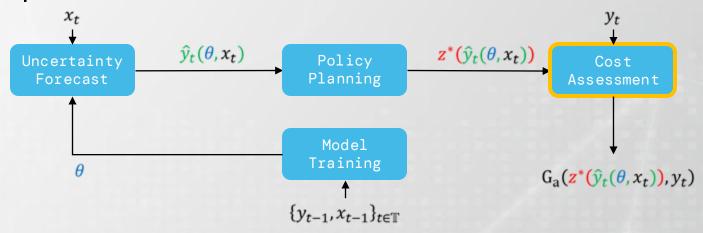
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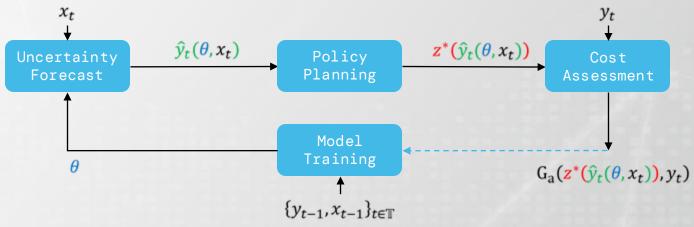


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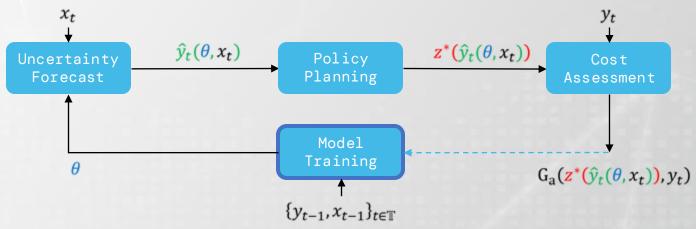


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Train Load Forecast and Reserve Model

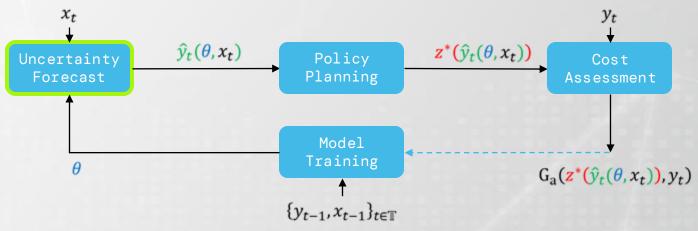
Forecast Loads and Reserve Requirements

Plan the Operation

$$\theta_{T} \in \underset{\theta \in \Theta, \hat{y}_{t}, z_{t}^{*}}{\operatorname{arg \, min}} \quad \frac{1}{T} \sum_{t \in \mathbb{T}} G_{a}(z_{t}^{*}, y_{t})$$

$$s.t. \quad \hat{y}_{t} = \Psi(\theta, x_{t}) \ \forall t \in \mathbb{T}$$

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Train Load Forecast and Reserve Model

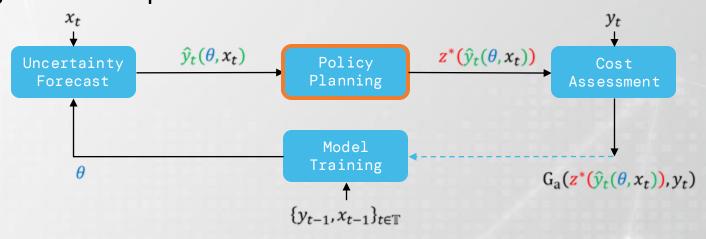
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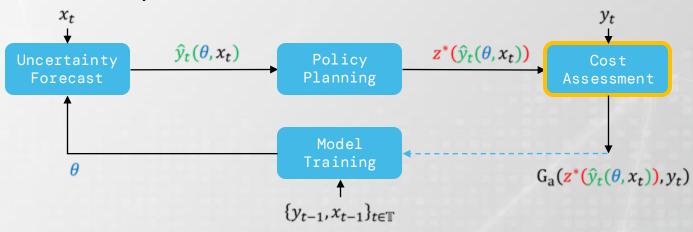
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Convergence

Assumptions

- Policy Planning has a unique solution. Define $\zeta(y) := \operatorname{argmin}_{z \in Z} G_p(z, y)$
- The set Z is non-empty and bounded
- Feasible set of the dual of Cost Assessment function is non-empty and bounded
- Forecasting function is continuous in both arguments

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- Forecast parameter are in Θ, which is compact
- Forecast target Y_t which is stationary, ergodic and integrable
- Context X_t is a measurable function of Y_t

Result

Hence
$$\lim_{T \to \infty} d(\theta_T, S^*) = 0$$
 wp1, for $S^* = \operatorname{argmin}_{\theta \in \Theta} \mathbb{E}\left[G_a\left(\zeta(\Psi(\theta, X)), Y\right)\right]$

For more

Application-Driven Learning: A Closed-Loop Prediction and Optimization Approach Applied to Dynamic Reserves and Demand Forecasting

Joaquim Dias Garcia 💿, Alexandre Street 💿, Tito Homem-de-Mello 💿, Francisco D. Muñoz 💿

Published Online: 9 Sep 2024 | https://doi.org/10.1287/opre.2023.0565



Solution method:

MTIP

$$\theta_T \in \operatorname*{arg\;min}_{\theta \in \Theta, \hat{y}_t, z_t^*}$$

$$\theta_T \in \underset{\theta \in \Theta, \hat{y}_t, z_t^*}{\operatorname{arg \, min}} \left| \frac{1}{T} \sum_{t \in \mathbb{T}} G_a(z_t^*, y_t) \right|$$

s.t.
$$\hat{y}_t = \Psi(\theta, x_t) \ \forall t \in \mathbb{T}$$

$$z_t^* \in \operatorname*{arg\,min}_{z \in Z} G_p(z, \hat{y}_t) \ \forall t \in \mathbb{T}$$





Linear models

$$G_{i}(z, y) = c_{i}^{\top} z + Q_{i}(z, y)$$

$$Q_{i}(z, y) = \min_{u} \{ q_{i}^{\top} u \mid W_{i} u \ge b_{i} - H_{i} z + F_{i} y \}$$



KKT based MPEC reformulation

$$\min_{\theta \in \Theta, \hat{y}_t, z_t^*, u_t, \pi_t}$$

$$\min_{\theta \in \Theta, \hat{y}_t, z_t^*, u_t, \pi_t} \frac{1}{T} \sum_{t \in \mathbb{T}} \left[c_a^\top z_t^* + Q_a(z_t^*, y_t) \right]$$

$$\forall t \in \mathbb{T}$$
:

$$\hat{y}_t = \Psi(\theta, x_t)$$

$$W_p y_t + H_p z_t^* \ge b_p + F_p \hat{y}_t$$

$$Az_t^* \geq h$$

$$W_p^{\mathsf{T}} \pi_t = q_p$$

$$H_p^\top \pi_t + A^\top \mu_t = c_p$$

$$\pi_t, \mu_t \geq 0$$

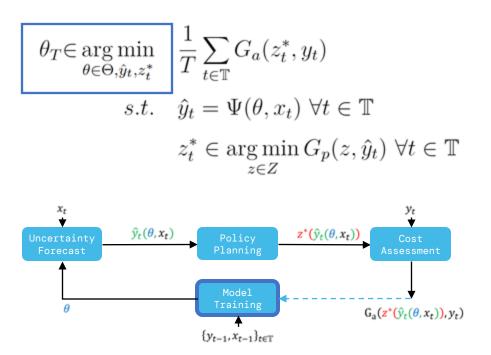
$$\pi_t \perp W_p u_t + H_p z_t^* - b_p - F_p \hat{y}_t$$

$$\mu_t \perp Az_t^* - h$$

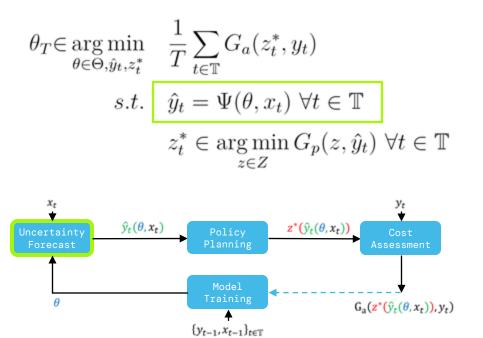
Solution method: MILP with BilevelJuMP.jl

```
\begin{aligned} \min_{\theta, \hat{y}_t, z_t, u_t^a, u_t} \quad & \frac{1}{T} \sum_{t \in \mathbb{T}} c_a^\top z_t^* + q_a^\top u_t^a \\ s.t. \quad & \forall t \in \mathbb{T} : \\ & C\theta \geq g \\ & \hat{y}_t = \theta^\top x_t \\ & W_a u_t^a \geq b_a - H_a z_t + F_a y_t \\ & z_t \in \mathop{\arg\min}_{z_t, u_t} c_p^\top z_t + q_p^\top u_t \\ & W_p u_t \geq b_p - H_p z_t + F_p \hat{y}_t \\ & A z_t \geq h \end{aligned}
```

```
m = BilevelModel()
@variable(Upper(m), 0[1:L])
@variable(Upper(m), y[1:M,1:T])
@variable(Lower(m), u[1:N,1:T])
@variable(Lower(m), z[1:P,1:T])
@variable(Upper(m), ua[1:0,1:T])
@objective(Upper(m),
    Min, 1/T * 5(ca'z[:,t] + qa'ua[:,t] for t \in T)
Qconstraint(Upper(m), C*\theta \ge q)
for t E T
@constraint(Upper(m),
    y_{hat}[:,t] = \theta'x[:,t]
@constraint(Upper(m),
    Wa * ua[:,t] \geq ba - Ha * z[:,t] + Fa * y[:,t])
end
@objective(Lower(m),
    Min, \sum (cp'z[:,t] + qp'u[:,t] for t \in T)
for t E T
@constraint(Lower(m),
    A * z[:,t] \ge h
@constraint(Lower(m),
    Wp * u[:,t] \geq bp - Hp * z[:,t] + Fp * y_hat[:,t])
end
```



```
Algorithm 1: Pseudo algorithm
   Result: Optimized \theta
   Initialize \theta:
    while Not converged do
        Update \theta:
        for t \in \mathbb{T} do
             Forecast: \hat{y}_t \leftarrow \Psi(\theta, x_t);
              Plan Policy: z_t^* \leftarrow \arg\min_{z \in Z} G_p(z, \hat{y}_t);
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        Compute cost: cost(\theta) \leftarrow \sum_{t \in \mathbb{T}} (cost_t)
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ApplicationDrivenLearning.jl

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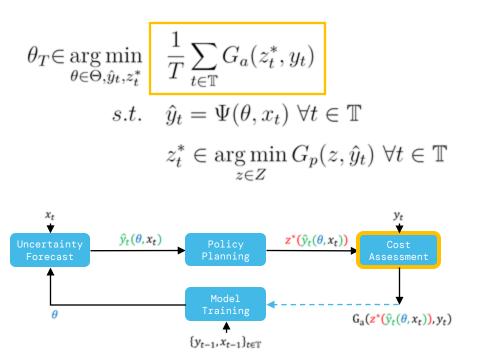
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$$y_t$$

$$y$$

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   end
```

Naïve choice of "Update heta" : zero order methods like Nelder Mead

Pros:

- Simplicity: forecast, planning and assess models can be basically anything as long as we can compute final costs efficiently.

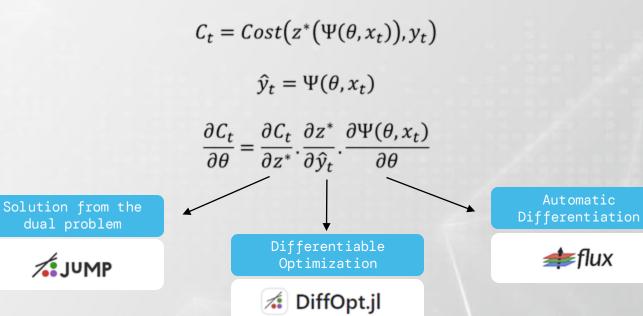
Cons:

- Search method for cost minimization not smartly guided by problem structure and can take long time.
- For high-cardinality model parameters, can become intractable.



JUMP

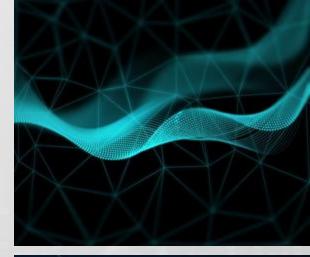
Another choice of "Update θ ": first order methods like Gradient Descent



The New Package

•The *ApplicationDrivenLearning.jl* package presents an easy way of training models in the closed loop fashion.

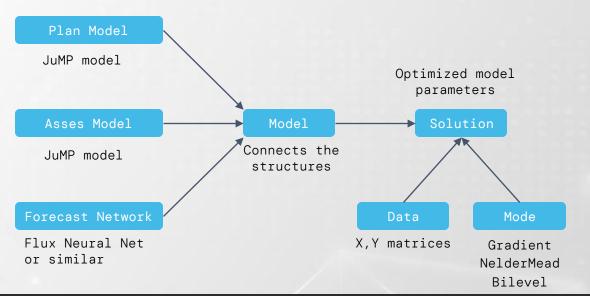


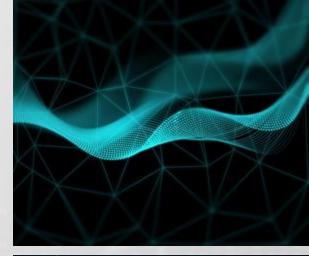




The Package

•The *ApplicationDrivenLearning.jl* package presents an easy way of training models in the closed loop fashion.







The Package

Solution methods

• Bilevel Optimization: JuMP.jl + BilevelJuMP.jl

Nelder Mead: JuMP.jl + Optim.jl

Gradient Descent: JuMP.jl + DiffOpt.jl + Flux.jl

· Problem classes

- · Linear & Quadratic
- Conic
- Non-Linear

Other features

- MPI parallelism
- Solver agnostic (use your favorite)



```
= rand(DiscreteUniform(0, 1), (30, 1)) .* 2
# main model and policy / forecast variables
model = ApplicationDrivenLearning.AppDrivenModel()
@variables(model, begin
    z, ApplicationDrivenLearning.Policy
    θ, ApplicationDrivenLearning.Forecast
end)
# plan model
@variables(ApplicationDrivenLearning.Plan(model), begin
    c1 ≥ 0
    c2 ≥ 0
end)
@constraints(ApplicationDrivenLearning.Plan(model), begin
    c1 ≥ 100 * (0.plan-z.plan)
    c2 ≥ 20 * (z.plan-θ.plan)
end)
@objective(ApplicationDrivenLearning.Plan(model), Min, 10*z.plan + c1 + c2)
# assess model
@variables(ApplicationDrivenLearning.Assess(model), begin
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www.soma.energy

```
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ApplicationDrivenLearning.jl

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# basic setting
set_optimizer(model, HiGHS.Optimizer)
set silent(model)
# forecast model
nn = Chain(Dense(1 => 1; bias=false))
ApplicationDrivenLearning.set forecast model(model, nn)
# training and getting solution
sol = ApplicationDrivenLearning.train!(
    model,
    Χ,
    ApplicationDrivenLearning.Options(
        ApplicationDrivenLearning.NelderMeadMode
sol.params
sol.cost / 30 | 38.666676f0
```



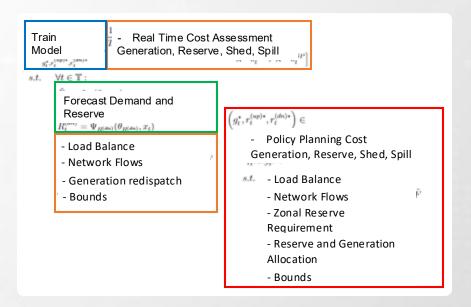
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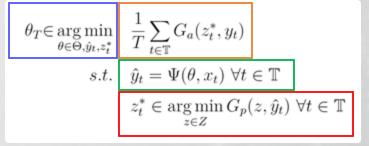


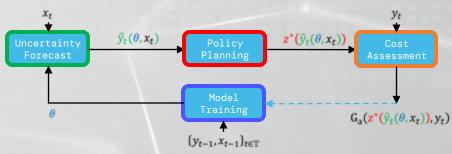
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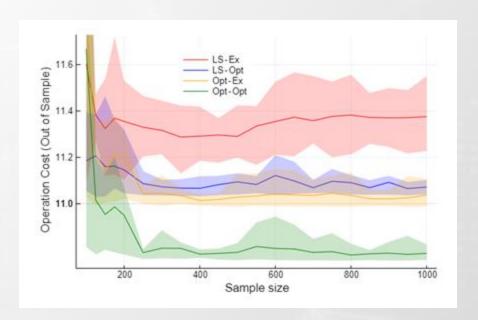
Application Driven Model for Load and Reserve





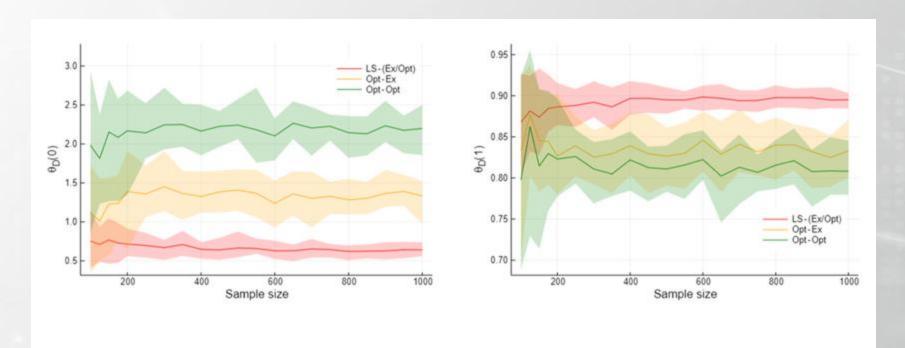


Case Study: Convergence of Objective



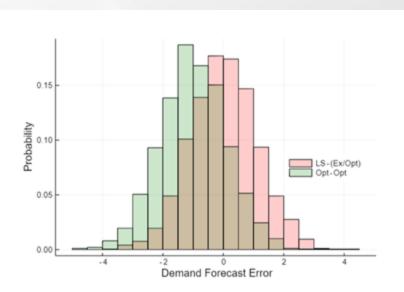
- LS-Ex (red)
 Least-Squares load
 Exogenous reserves
- LS-Opt (blue)
 Least-Squares load
 Optimized reserves
- Opt-Ex (yellow)
 Optimized load
 Exogenous reserves
- Opt-Opt (green)
 Optimized load
 Optimized reserves

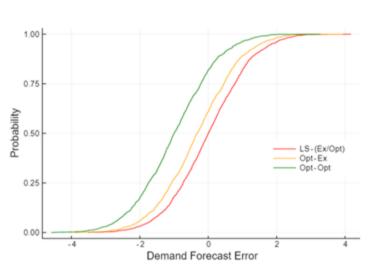
Case Study: Convergence of Solution





Case Study: Biased forecast





Case Study: Multiple methods

N Buses	LS	MPI-NM	MPI-GD	GD
24	7.608,62	7.617,16 (-0,11%)	7.627,59 (-0,25%)	7.604,44 (0,05%)
118	11.515,94	11.177,89 (2,94%)	10.572,90 (8,19%)	10.579,62 (8,13%)
179	123.313,02	113.038,81 (8,33%)	82.549,92 (33,06%)	90.703,52 (26,44%)
240	801.135,26	-	760.829,71 (5,03%)	767.958,60 (4,14%)
300	82.968,84	-	$76.625,42 \ (7,65\%)$	76.313,26 (8,02%)
500	29.154,24	-	$29.051,50 \ (0,35\%)$	29.061,10 (0,32%)
588	33.496,56	-	29.362,07 (12,34%)	30.577,54 (8,71%)
793	48.403,34	-	37.940,31 (21,62%)	42.558,32 (12,08%)
1354	176.562,56	-	172.565,10 (2,26%)	-



The end

Package:

- github.com/LAMPSPUC/ApplicationDrivenLearning.jl
- Friendly interface
- Multiple solution methods and solvers
- HPC Ready

Method:

- Outperformed open-loop framework
- Optimal forecasts are BIASED
- Meaningful improvements even in very large-scale systems
- Can be used in practice

