# QUBO.jl

*A Julia ecosystem for*
*Quadratic Unconstrained Binary Optimization*

**Pedro Maciel Xavier**

Purdue University
Federal University of Rio de Janeiro

# Summary

*What has to be done to incorporate Quantum & other Physics-inspired Technologies in Optimization Workflows?*

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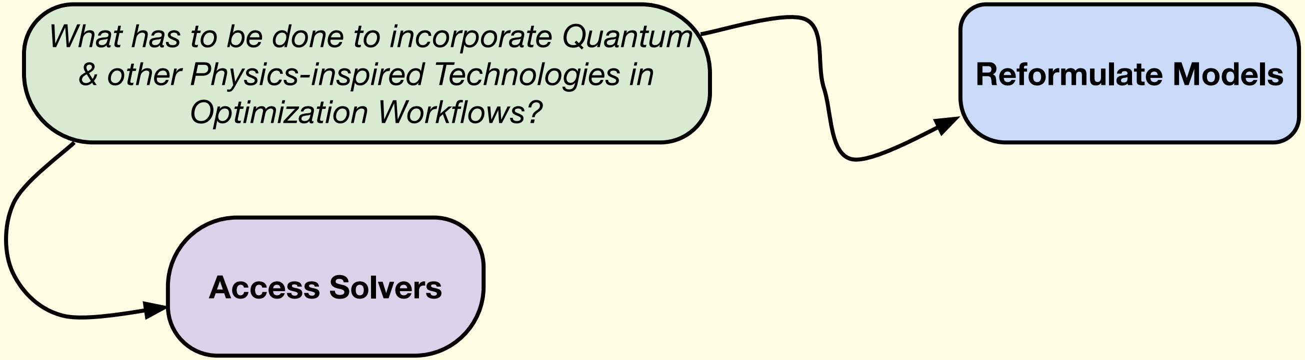
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Reformulate Models
Access Solvers

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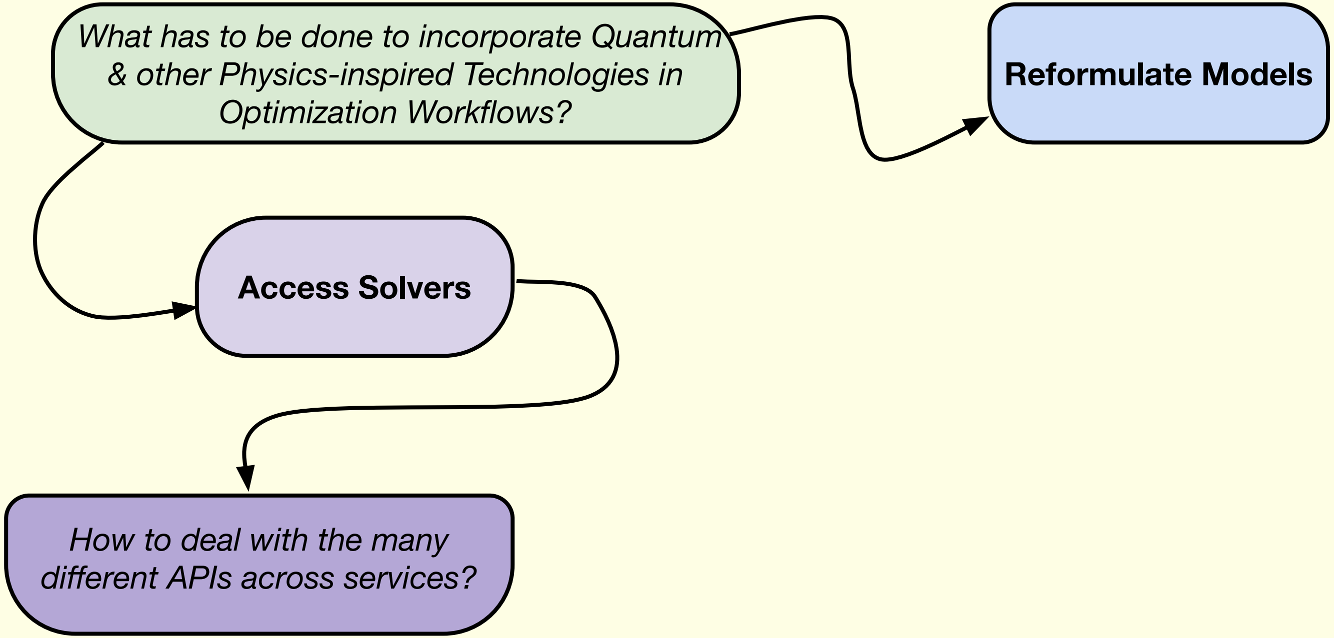
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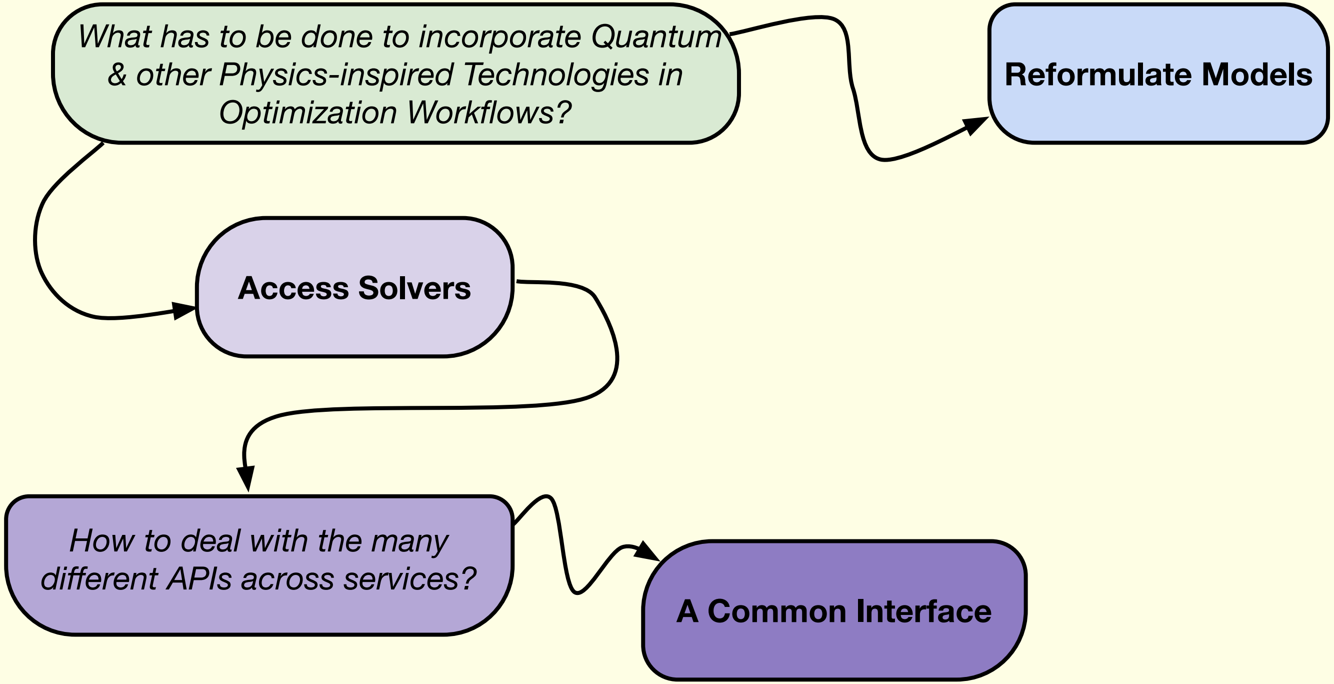
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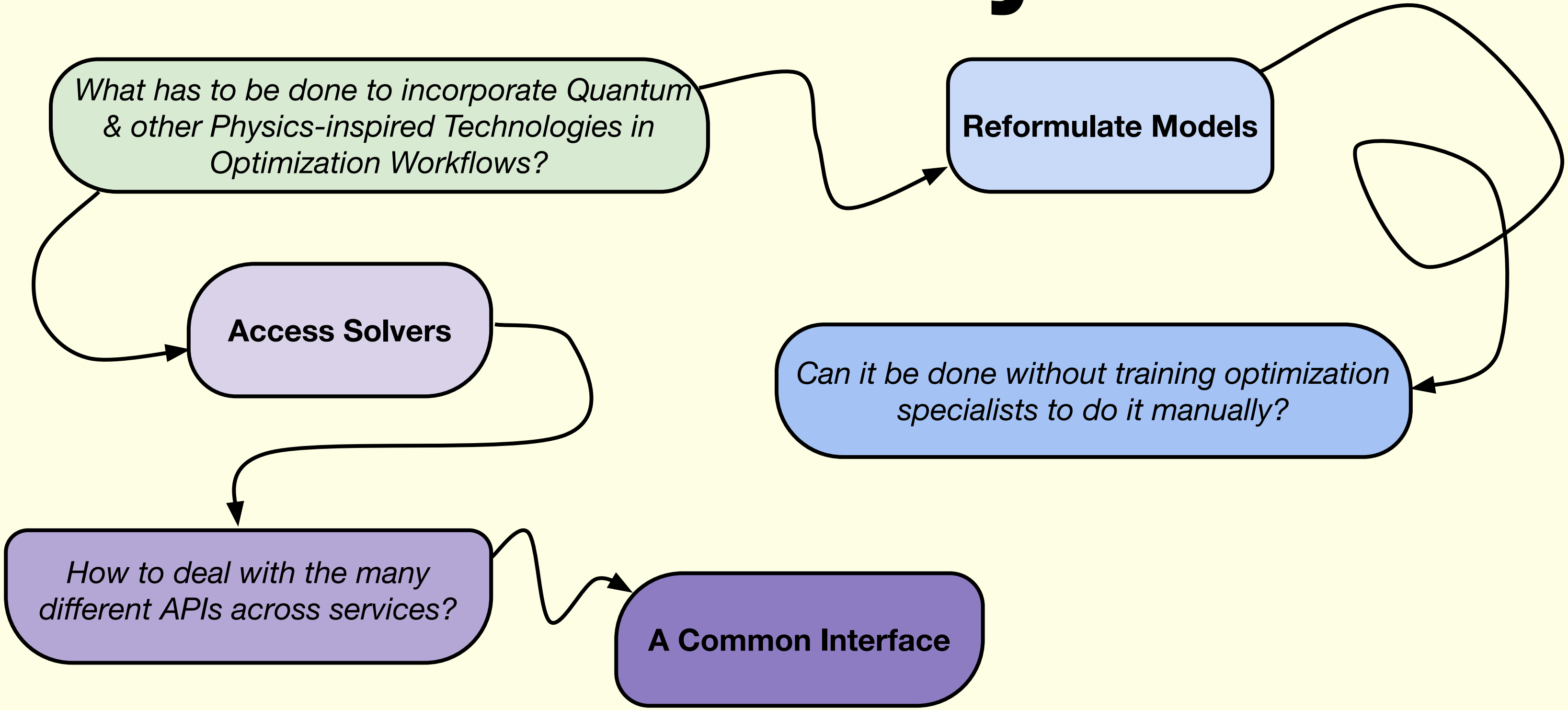
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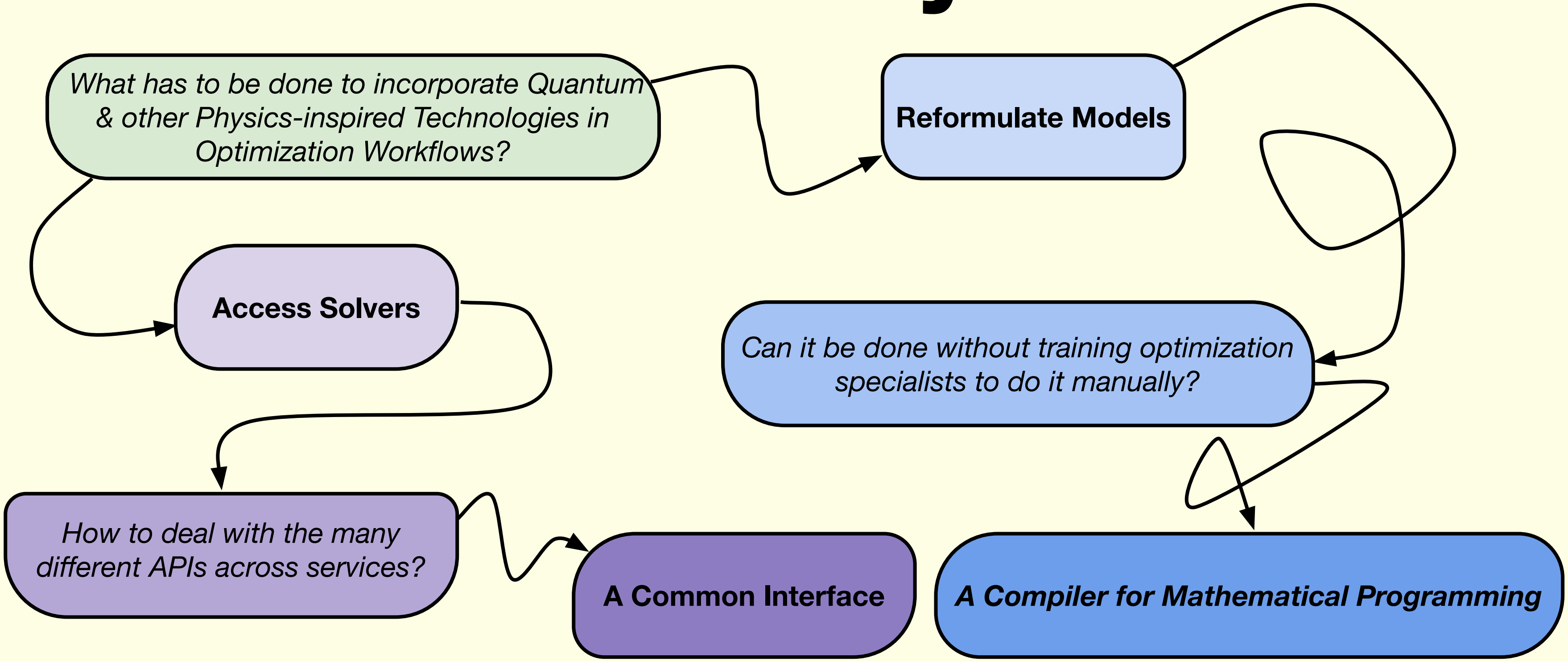
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A Common Interface
A Compiler for Mathematical Programming

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**A Common Interface**

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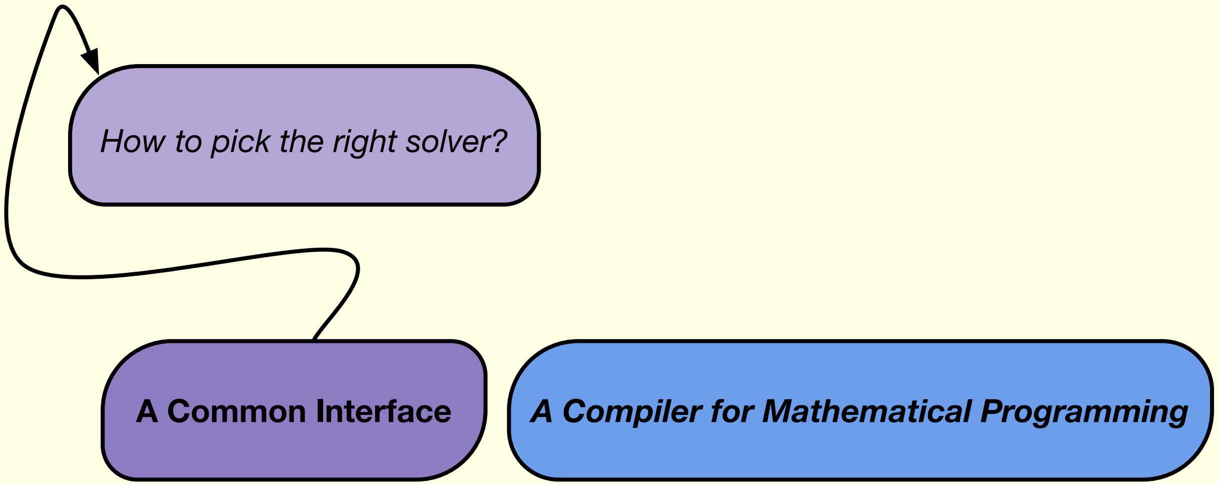
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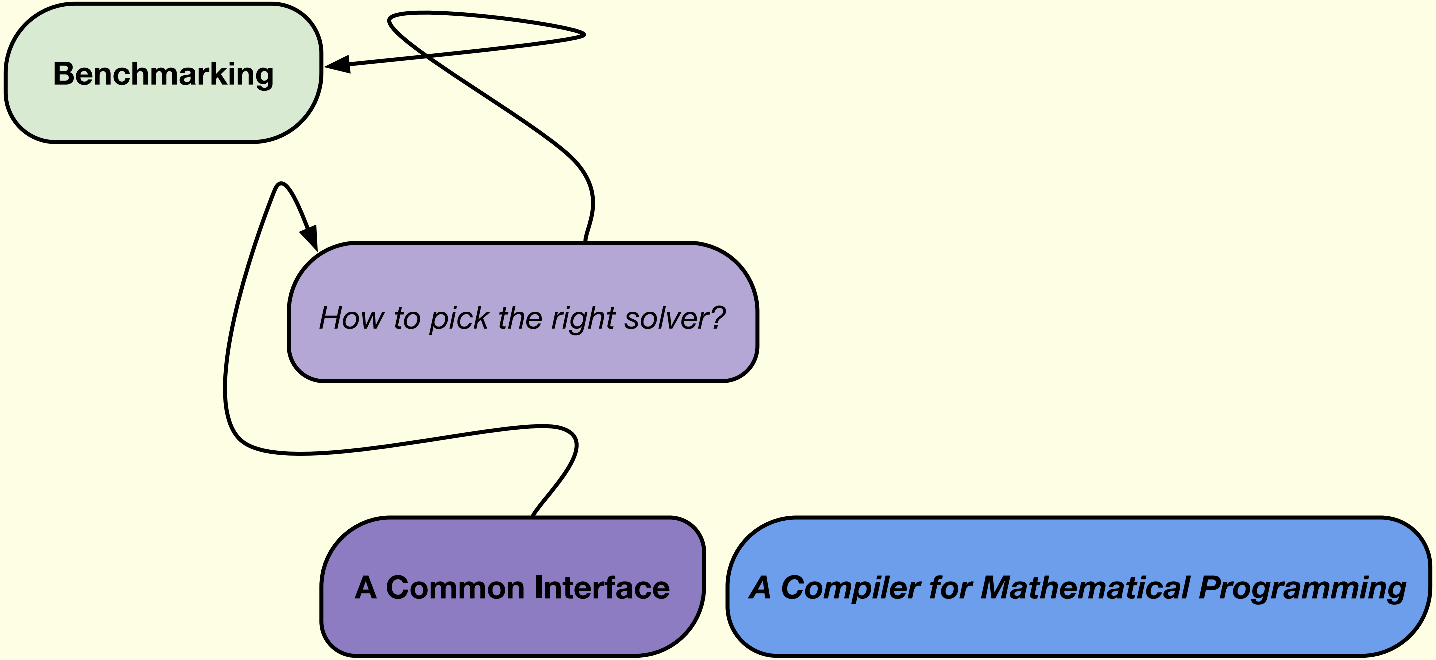

Benchmarking
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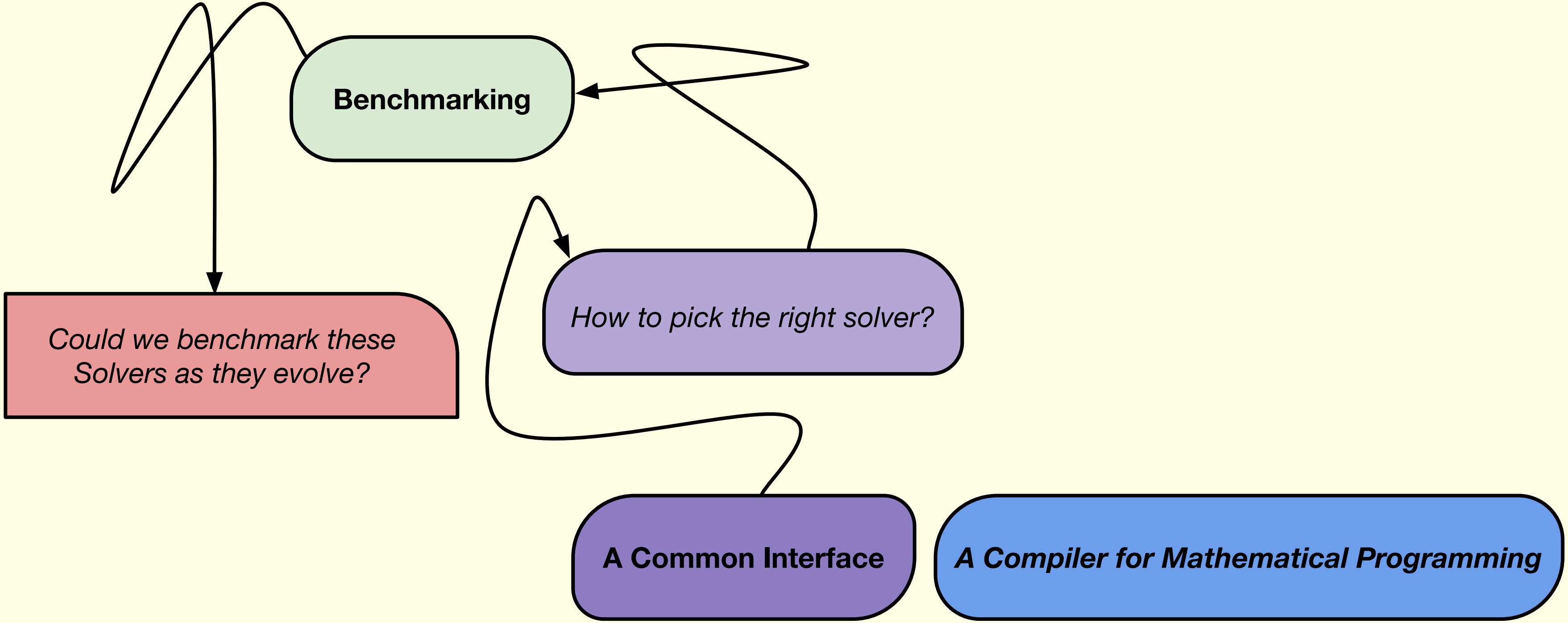

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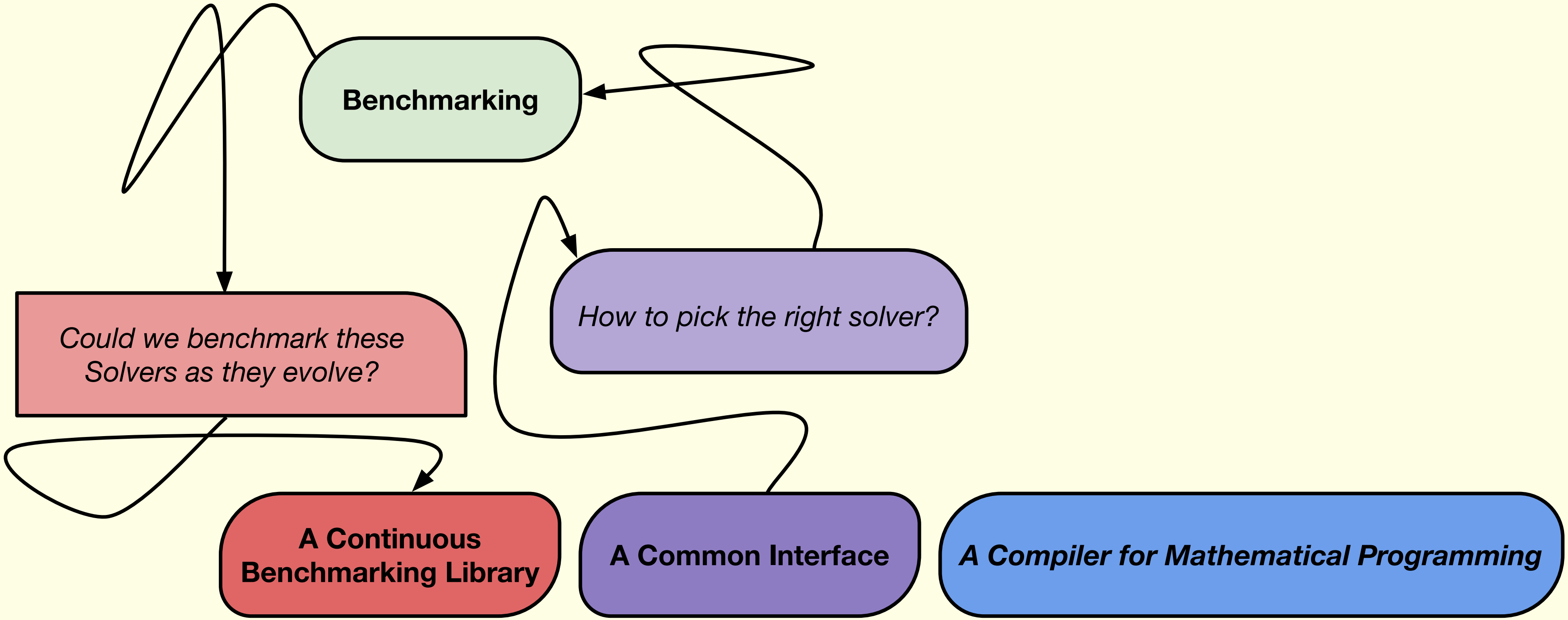

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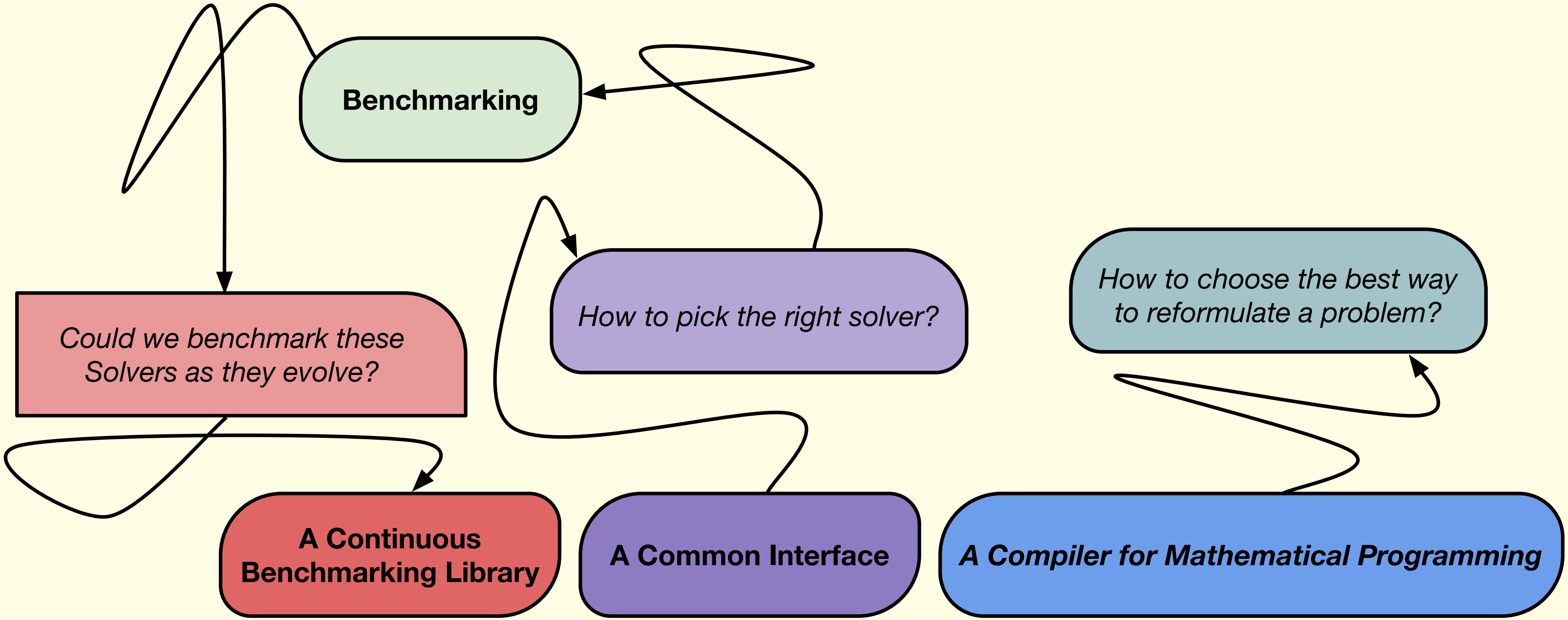
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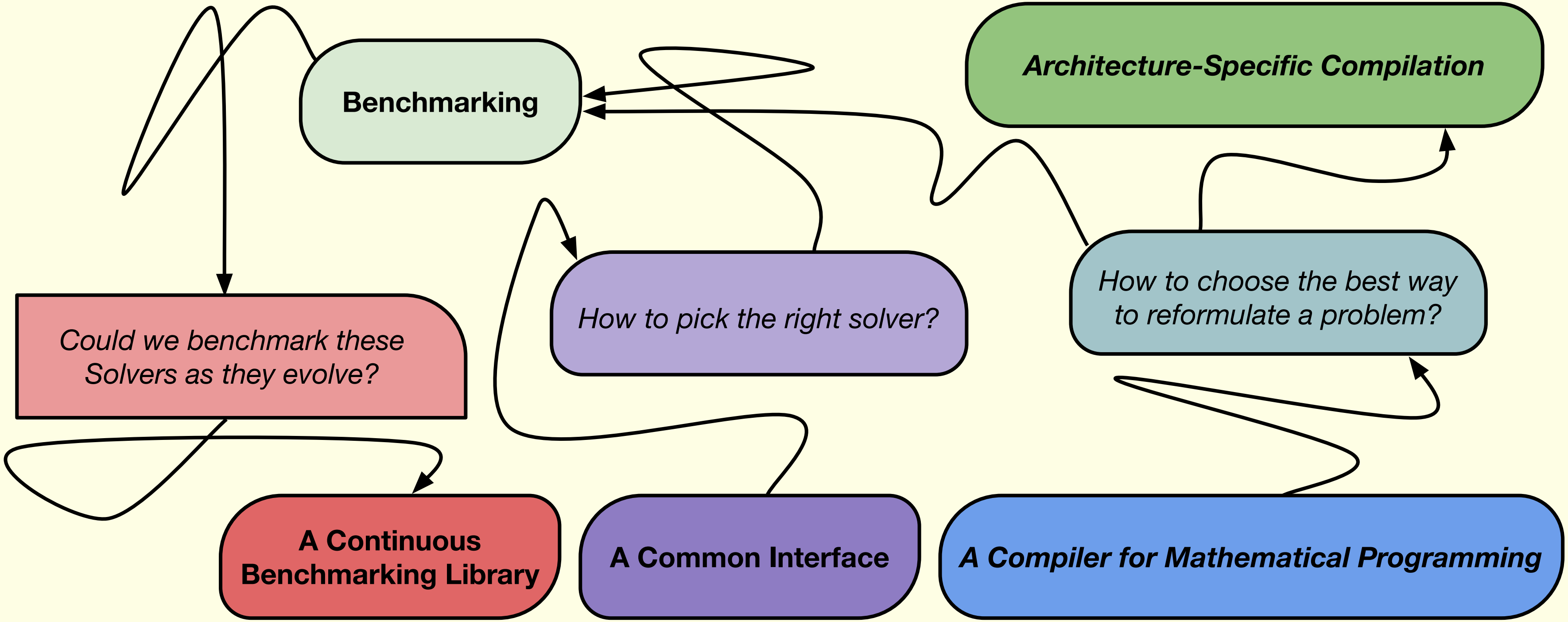
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A Continuous Benchmarking Library

A Common Interface

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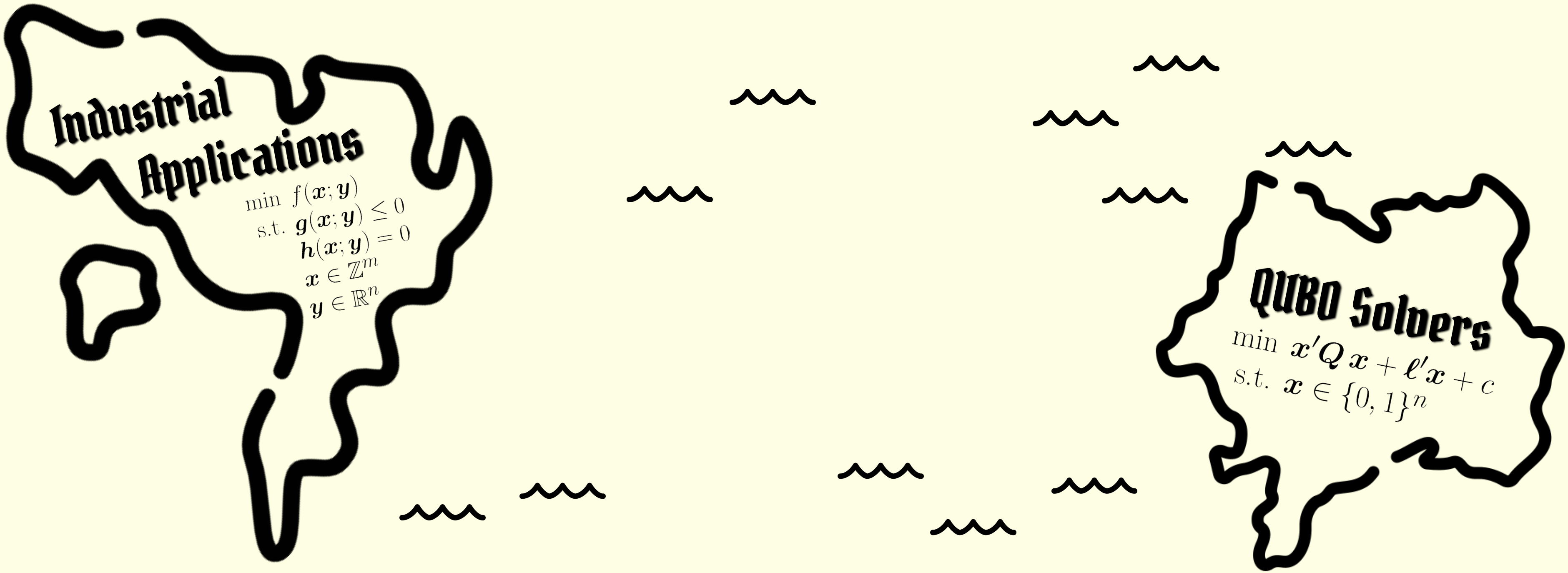

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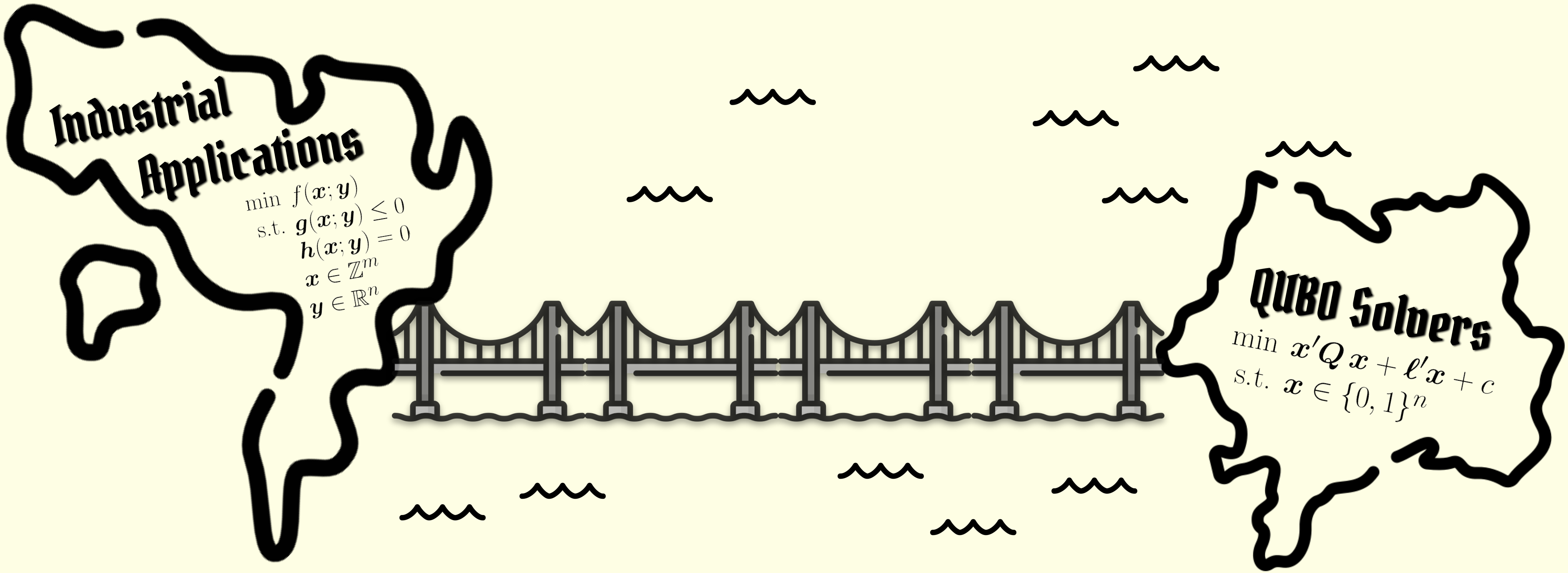
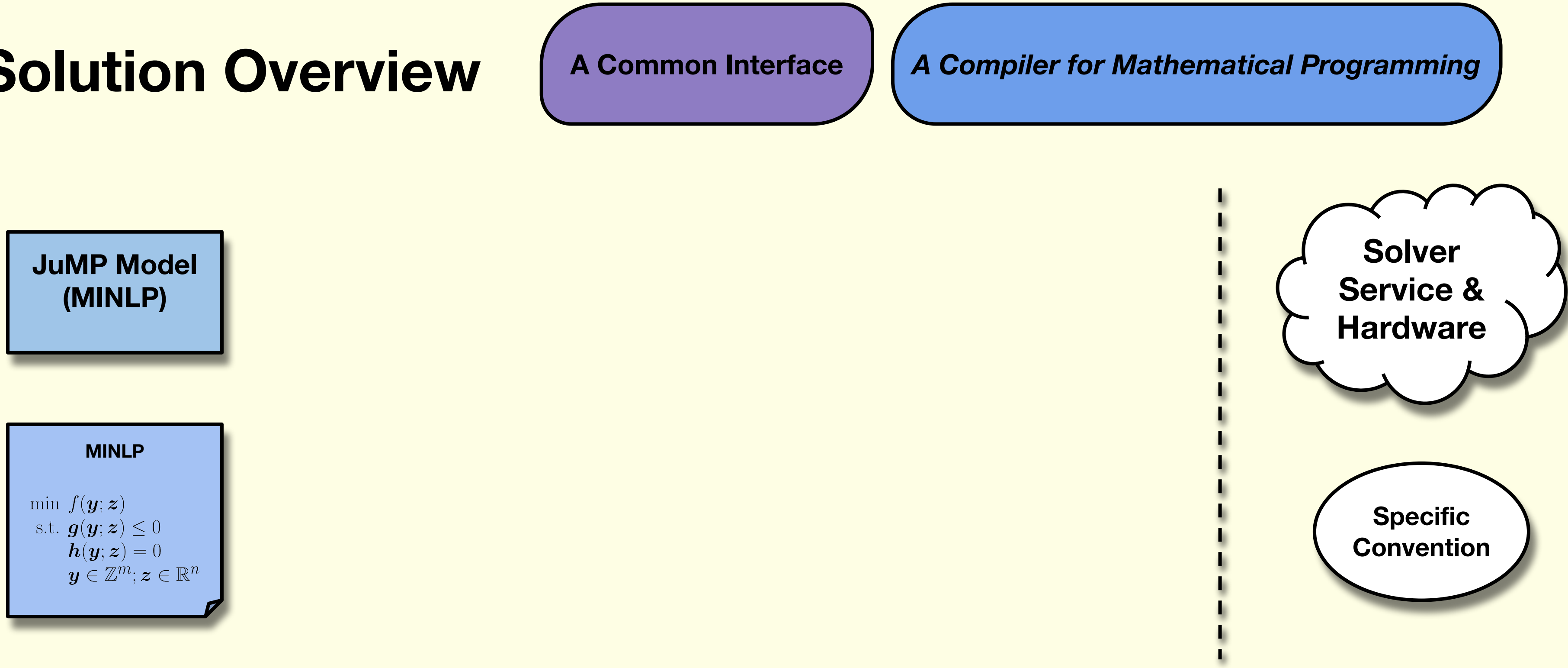


# Solution Overview



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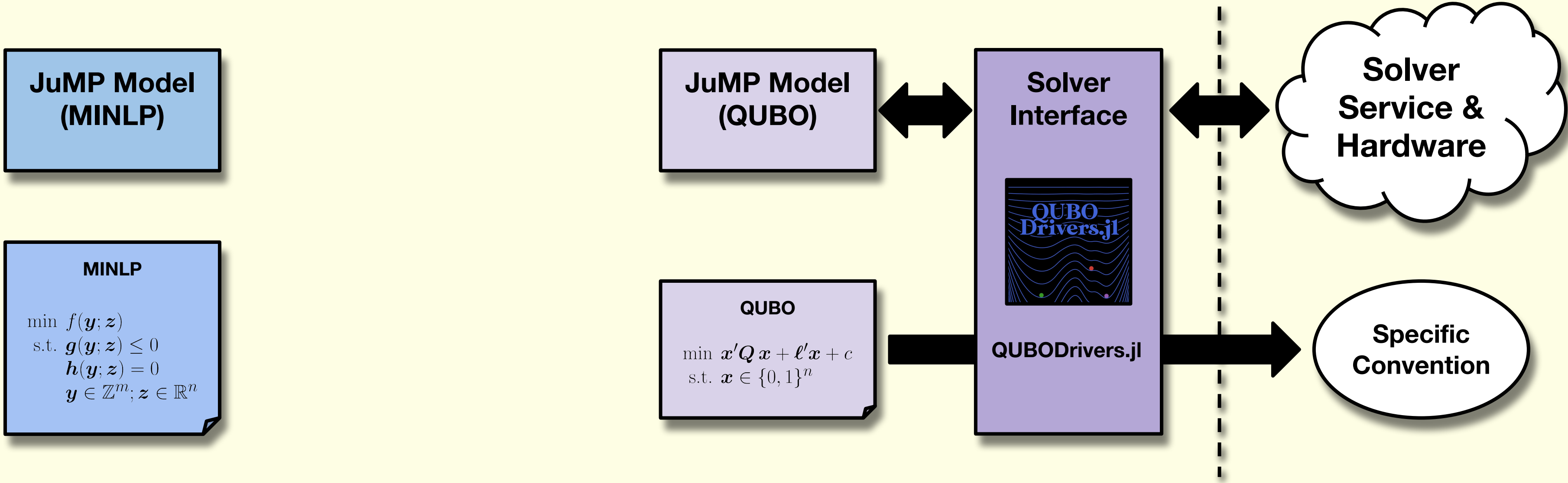

# Solution Overview

A Common Interface

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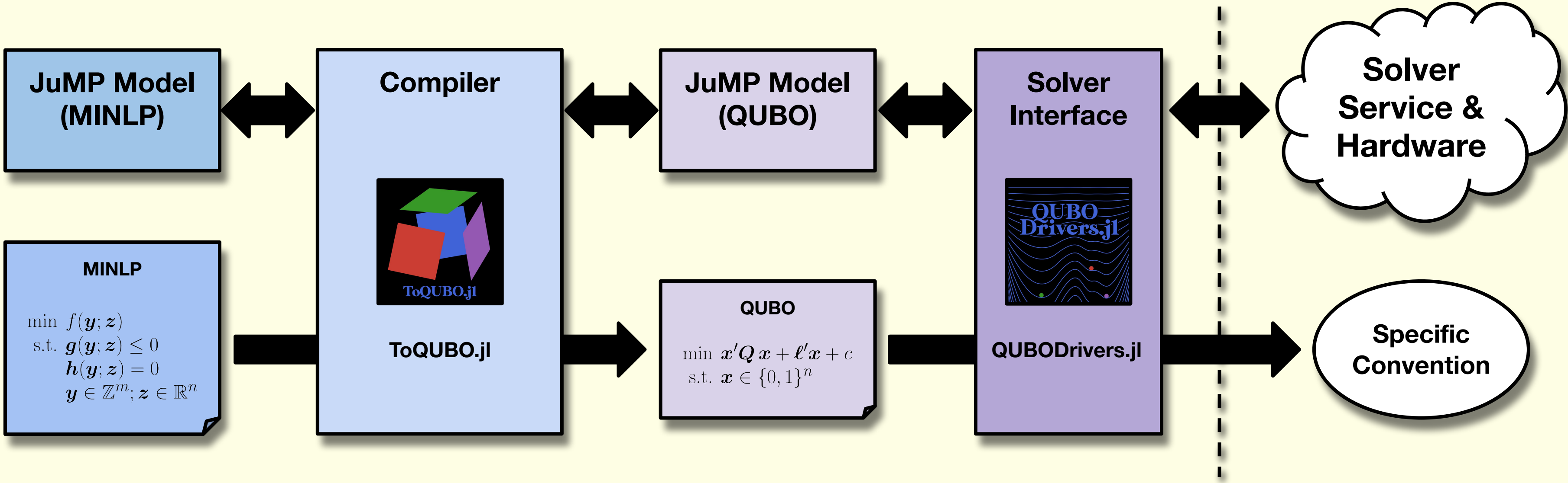
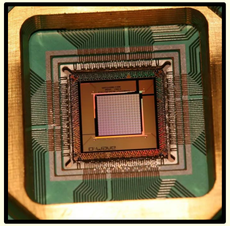
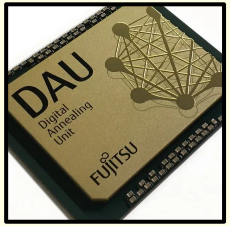


# Integrating an heterogeneous Solver Landscape

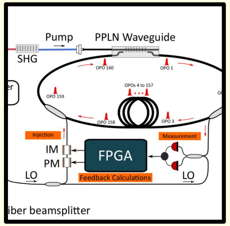


D-Wave

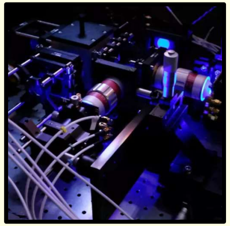



Fujitsu

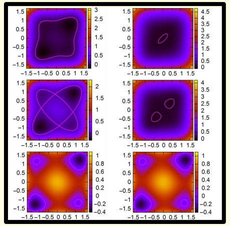
IBM




P. L. McMahon et al., 2016

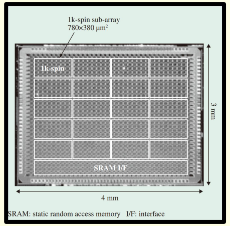


Microsoft Research



Toshiba, Goto et al., 2019

**Quantum Annealing, Digital Annealing, Variational Quantum Eigensolver, Quantum Alternating Optimization Ansatz, Coherent Ising Machine, Analog Iterative Machine, Simulated Bifurcation Machine, CMOS Annealing…**




Hitachi, Yamaoka et al.

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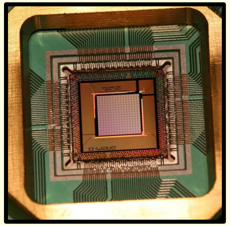
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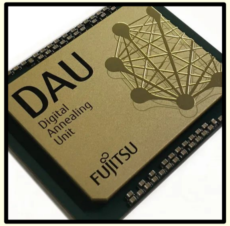
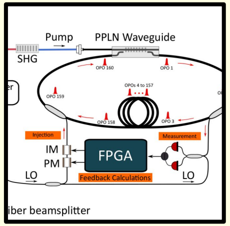


Fujitsu


IBM



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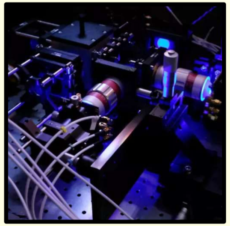
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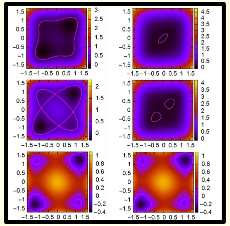
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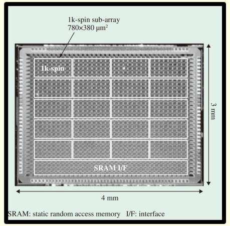
Julia Mathematical Programming



Hitachi, Yamaoka et al.

# A Common Solver Interface

```julia
using JuMP
using QiskitOpt # IBM Qiskit Optimizati

model = Model(QiskitOpt.QAOA.Optimizer)

@variable(model, x[1:n], Bin)
@objective(
  model,
  Min,
  x' * Q * x + ℓ' * x + c
)

optimize!(model)

@show objective_value(model)
@show value.(x)
```

```julia
using JuMP
using DWave # DWave Quantum Annealing

model = Model(DWave.Optimizer)

@variable(model, x[1:n], Bin)
@objective(
  model,
  Min,
  x' * Q * x + ℓ' * x + c
)

optimize!(model)

@show objective_value(model)
@show value.(x)
```

```julia
ing JuMP
ing PySA # NASA Parallel Tempering

del = Model(PySA.Optimizer)

ariable(model, x[1:n], Bin)
bjective(
model,
Min,
x' * Q * x + ℓ' * x + c

timize!(model)

how objective_value(model)
how value.(x)
```

# Testing and Benchmarking Solvers

A Continuous Benchmarking Library

QUBO Lib

# Testing and Benchmarking Solvers

A Continuous Benchmarking Library

| Sources Summary | | |
|---|---|---|
| **Collection** | **Instances** | **Size Range** |
| arXiv:2103.008464 (3R3X) | 2300 | 16 - 4096 |
| arXiv:1903.100928 (3R3X) | 3200 | 16 - 4096 |
| arXiv:1903.100928 (5R5X) | 307 | 24 - 24576 |
| qplib* | 23 | 120 - 1225 |

*QPLIB: A Library of Quadratic Programming Instances, Mathematical Programming Computation, 2018

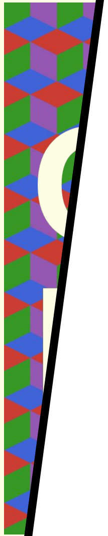
# Testing and Benchmarking Solvers

**A Continuous Benchmarking Library**

```julia
QUBOLib.load_index() do index
    db = QUBOLib.database(index)
    df = DBInterface.execute(
        db,
        """
        SELECT instance FROM Instances
        WHERE dimension < 100 AND quadratic_density < 0.5;
        """,
    ) |> DataFrame

    codes = collect(Int, df[!, :instance])

    @info "Running DWave Neal"
    QUBOLib.run!(
        index, DWave.Neal.Optimizer, codes; solver = "dwave-neal"
    )

    @info "Running DWave (Quantum)"
    QUBOLib.run!(
        index, DWave.Optimizer, codes; solver = "dwave"
    )
end
```

# Testing and Benchmarking Solvers

A Continuous ...ing Library



{ ISMP 2024 }

**Extensions of Integer Programming**
**Monday, 16:20-17:50**

# A Compiler for Mathematical Programming

$$\begin{aligned} \min \quad & f(\boldsymbol{y}; \boldsymbol{z}) \\ \text{s.t.} \quad & g_i(\boldsymbol{y}; \boldsymbol{z}) \in S_i \\ & \boldsymbol{y} \in \mathbb{Z}^m; \boldsymbol{z} \in \mathbb{R}^n \end{aligned}$$

# A Compiler for Mathematical Programming

$$\begin{array}{rl} \min & f(\boldsymbol{y}; \boldsymbol{z}) \\ \text{s.t.} & g_i(\boldsymbol{y}; \boldsymbol{z}) \in S_i \\ & \boldsymbol{y} \in \mathbb{Z}^m; \boldsymbol{z} \in \mathbb{R}^n \end{array}$$

**VARIABLE ENCODING**

$$\begin{array}{rl} \min & f(\boldsymbol{x}) \\ \text{s.t.} & g_i(\boldsymbol{x}) \in S_i \\ & \boldsymbol{x} \in \{0, 1\}^k \end{array}$$

# A Compiler for Mathematical Programming

$$\begin{aligned} \min \quad & f(\boldsymbol{y}; \boldsymbol{z}) \\ \text{s.t.} \quad & g_i(\boldsymbol{y}; \boldsymbol{z}) \in S_i \\ & \boldsymbol{y} \in \mathbb{Z}^m; \boldsymbol{z} \in \mathbb{R}^n \end{aligned}$$

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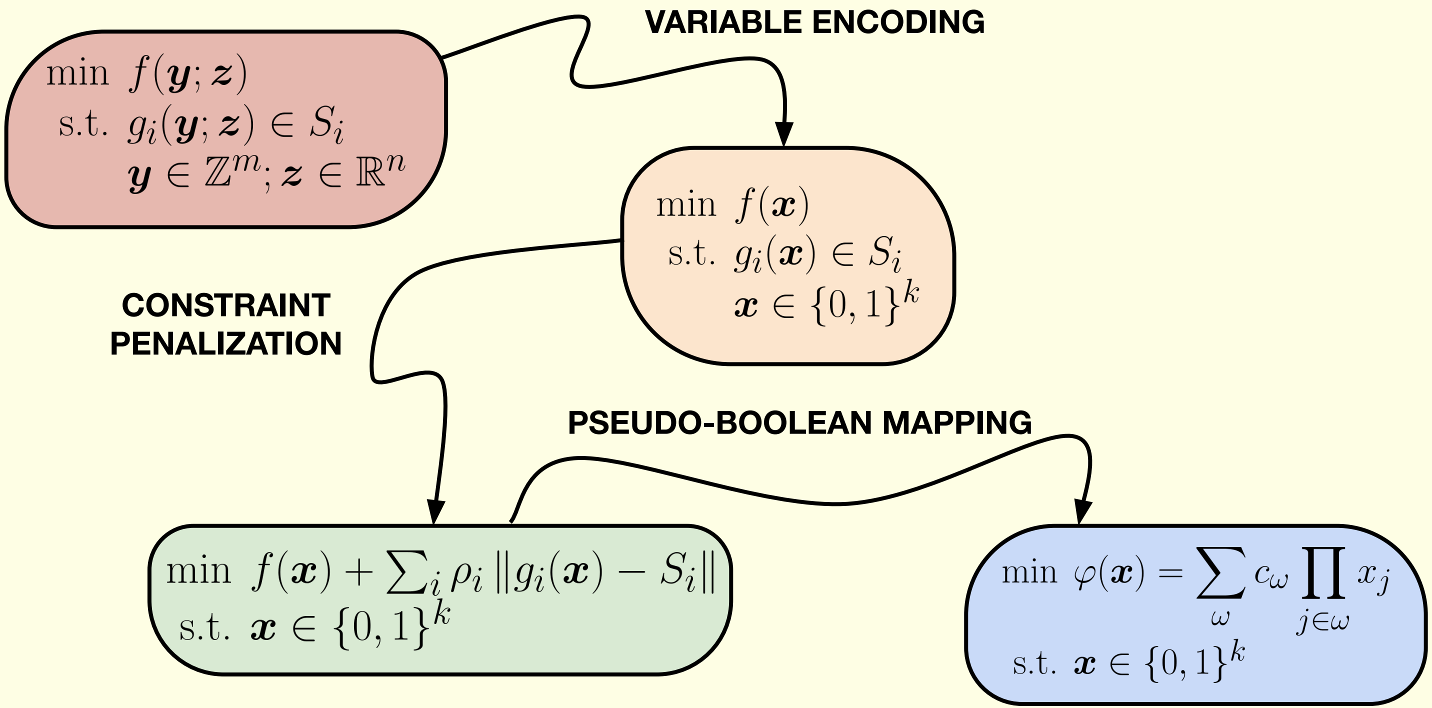
$$\begin{aligned} \min \quad & f(\boldsymbol{x}) \\ \text{s.t.} \quad & g_i(\boldsymbol{x}) \in S_i \\ & \boldsymbol{x} \in \{0, 1\}^k \end{aligned}$$

**CONSTRAINT PENALIZATION**

$$\begin{aligned} \min \quad & f(\boldsymbol{x}) + \textstyle\sum_i \rho_i \left\| g_i(\boldsymbol{x}) - S_i \right\| \\ \text{s.t.} \quad & \boldsymbol{x} \in \{0, 1\}^k \end{aligned}$$

# A Compiler for Mathematical Programming



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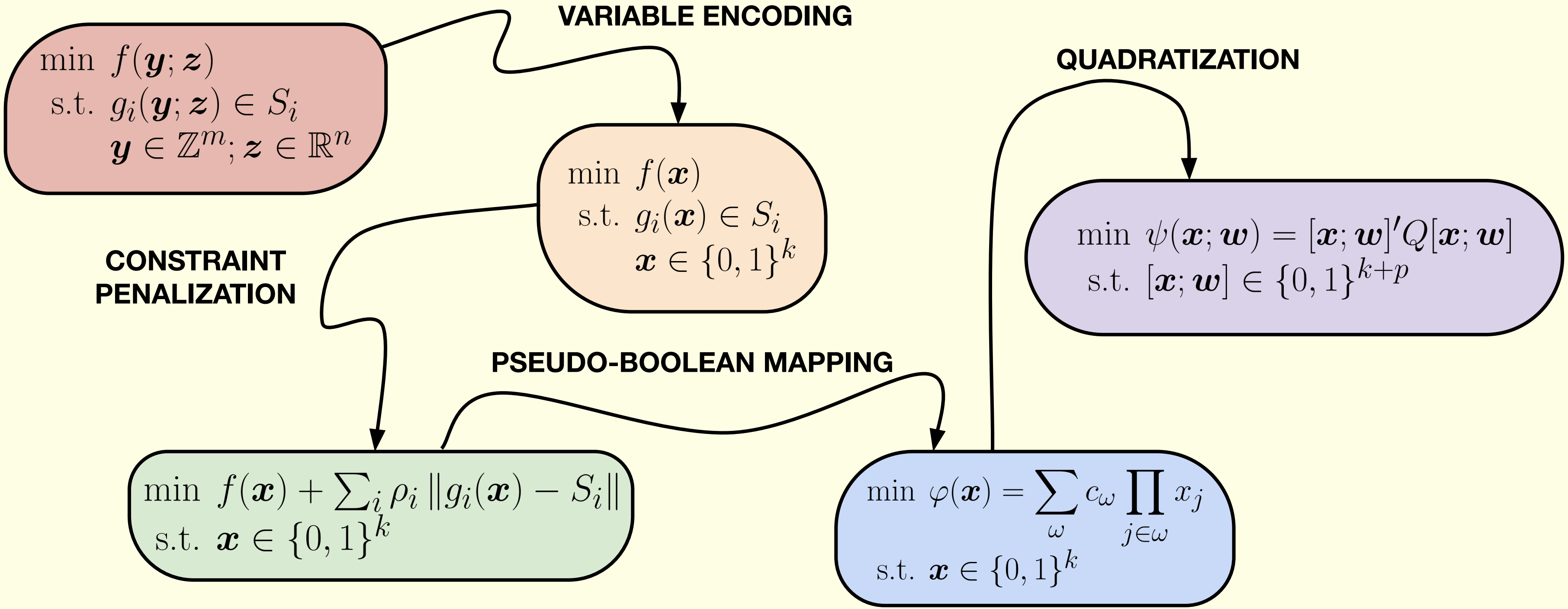


# A Compiler for Mathematical Programming

C, C++, Julia, Rust…

AMPL, JuMP, Pyomo…

# A Compiler for Mathematical Programming

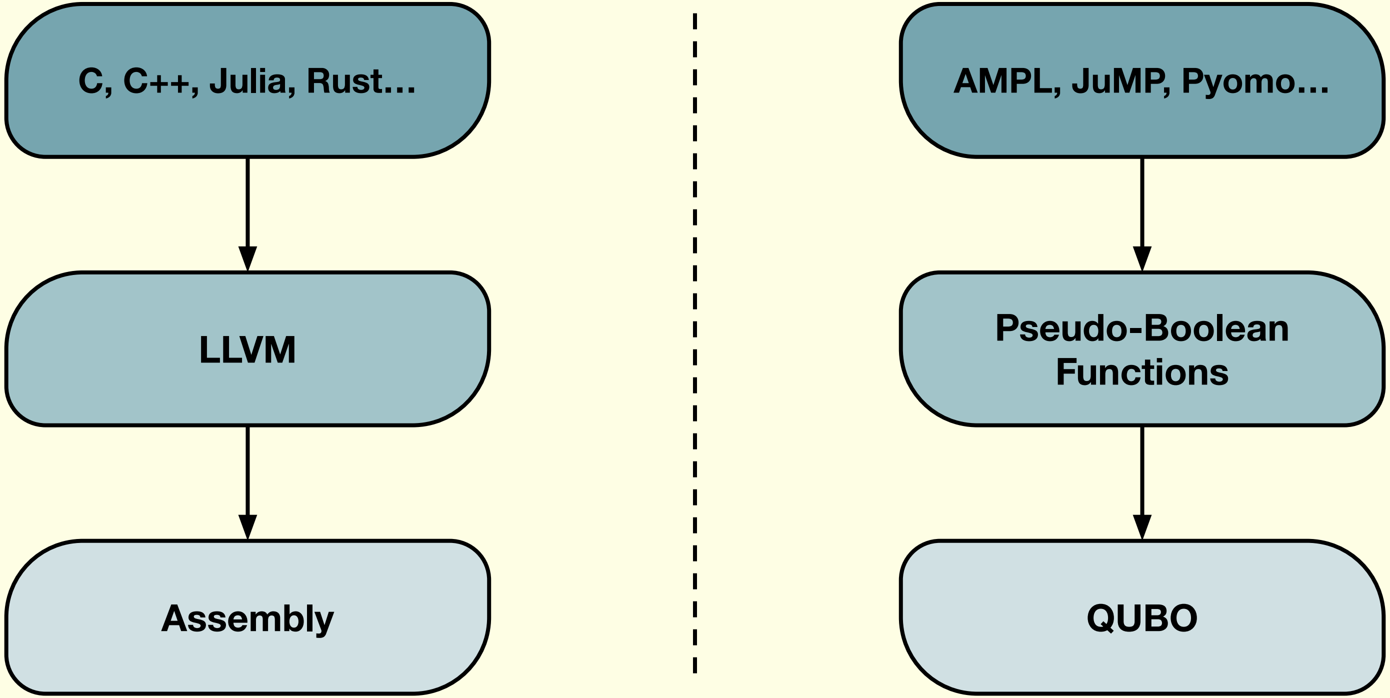
C, C++, Julia, Rust…

Assembly

AMPL, JuMP, Pyomo…

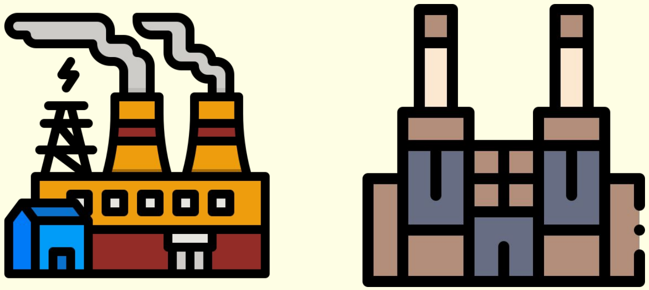
QUBO

# A Compiler for Mathematical Programming



# *EXAMPLE*

# Compilation use case: Generation capacity expansion



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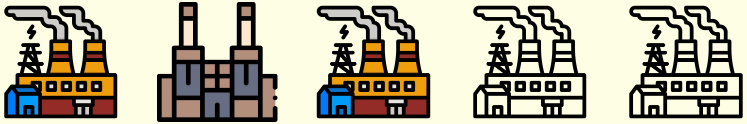
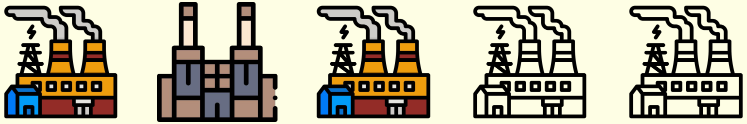


$$
\begin{aligned}
\min_{\mathbf{g},\mathbf{u},\mathbf{x}} \sum_t \mathbf{c}'\mathbf{g}^{(t)} + \mathbf{i}'\mathbf{x} & \\
\text{s.a.} \sum_j g_j^{(t)} = d^{(t)} & \qquad \forall t \\
g_j^{(t)} \leq u_j^{(t)} G_j^{(\max)} & \qquad \forall j, t \\
u_j^{(t)} \leq x_j & \qquad \forall j, t \\
g_j^{(t)} \in [0, G_j^{(\max)}] & \qquad \forall j, t \\
u_j^{(t)} \in \{0, 1\} & \qquad \forall j, t \\
x_j \in \{0, 1\} & \qquad \forall j
\end{aligned}
$$

# Compilation use case: Generation capacity expansion



OPERATION

$$\min_{\mathbf{g},\mathbf{u},\mathbf{x}} \sum_t \mathbf{c}'\mathbf{g}^{(t)} + \mathbf{i}'\mathbf{x}$$

$$\text{s.a.} \sum_j g_j^{(t)} = d^{(t)} \qquad \forall t$$

BALANCE

$$g_j^{(t)} \leq u_j^{(t)} G_j^{(\max)} \qquad \forall j, t$$

$$u_j^{(t)} \leq x_j \qquad \forall j, t$$

$$g_j^{(t)} \in [0, G_j^{(\max)}] \qquad \forall j, t$$

OPERATION

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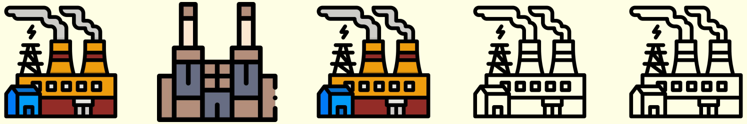

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UNIT COMMITMENT

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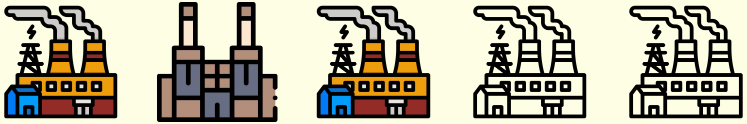

INVESTMENT

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INVESTMENT

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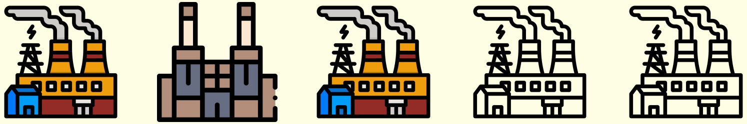

OPERATION INVESTMENT

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INVESTMENT

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OPERATION — INVESTMENT

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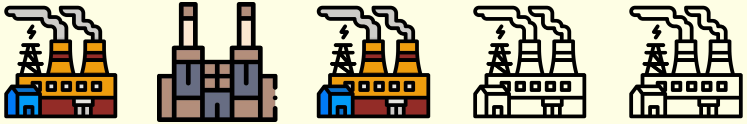
INVESTMENT

**REFORMULATION** →



$$\rho_{balance} \left( \sum_t \left( \sum_j g_j^{(t)} - d^{(t)} \right)^2 \right)$$

$$\rho_{invest} \left( \sum_{j,t} \left( g_j^{(t)} + s_{UB} - x_j G_j^{(max)} \right)^2 \right)$$

$$g_j^{(t)} = \alpha \sum_k 2^k y_{k,j}^{(t)} \quad \text{s.t.} \quad y_{k,j}^{(t)} \in \{0, 1\}$$

# Compilation use case: Generation capacity expansion

OPERATION | INVESTMENT

$$\min_{\mathbf{g},\mathbf{u},\mathbf{x}} \sum_t \mathbf{c}'\mathbf{g}^{(t)} + \mathbf{i}'\mathbf{x}$$

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$$u_j^{(t)} \leq x_j \quad \forall j, t$$

INVESTMENT

$$g_j^{(t)} \in [0, G_j^{(\max)}] \quad \forall j, t$$

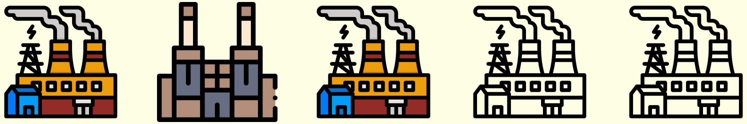
OPERATION

$$u_j^{(t)} \in \{0, 1\} \quad \forall j, t$$

UNIT COMMITMENT

$$x_j \in \{0, 1\} \quad \forall j$$

INVESTMENT



```julia
using JuMP
using PySA

model = Model(PySA.Optimizer)

@variable(model, 0 ≤ g[1:T,j=1:n] ≤ Gmax[j])
@variable(model, u[1:T,1:n], Bin)
@variable(model, x[1:n], Bin)

@objective(model, Min, sum(c'g[t,:] for t=1:T) + i'x)

@constraint(model, [t=1:T], sum(g[t,:]) == d[t])
@constraint(model, [t=1:T,j=1:n], g[t,j] ≤ u[t,j] * Gmax[j])
@constraint(model, [t=1:T,j=1:n], u[t,j] ≤ x[j])

optimize!(model)

@show objective_value(model)
@show value.(x)
```

# Compilation use case: Generation capacity expansion

OPERATION | INVESTMENT

$$\min_{\mathbf{g},\mathbf{u},\mathbf{x}} \sum_t \mathbf{c}'\mathbf{g}^{(t)} + \mathbf{i}'\mathbf{x}$$

$$\text{s.a.} \sum_j g_j^{(t)} = d^{(t)} \quad \forall t$$

BALANCE

$$g_j^{(t)} \leq u_j^{(t)} G_j^{(\max)} \quad \forall j, t$$

UNIT COMMITMENT

$$u_j^{(t)} \leq x_j \quad \forall j, t$$

INVESTMENT

$$g_j^{(t)} \in [0, G_j^{(\max)}] \quad \forall j, t$$

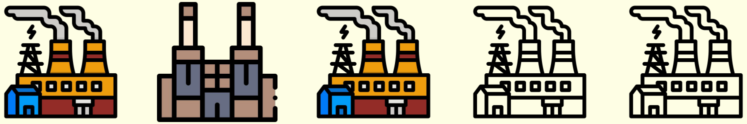
OPERATION

$$u_j^{(t)} \in \{0, 1\} \quad \forall j, t$$

UNIT COMMITMENT

$$x_j \in \{0, 1\} \quad \forall j$$

INVESTMENT



```julia
using JuMP
using PySA

model = Model(PySA.Optimizer)

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@variable(model, u[1:T,1:n], Bin)
@variable(model, x[1:n], Bin)

@objective(model, Min, sum(c'g[t,:] for t=1:T) + i'x)

@constraint(model, [t=1:T], sum(g[t,:]) == d[t])
@constraint(model, [t=1:T,j=1:n], g[t,j] <= u[t,j] * Gmax[j])
@constraint(model, [t=1:T,j=1:n], u[t,j] <= x[j])

optimize!(model)

@show objective_value(model)
@show value.(x)
```

# Compilation use case: Generation capacity expansion

$$\min_{\mathbf{g},\mathbf{u},\mathbf{x}} \underbrace{\sum_t \mathbf{c}'\mathbf{g}^{(t)}}_{\text{OPERATION}} + \underbrace{\mathbf{i}'\mathbf{x}}_{\text{INVESTMENT}}$$

$$\text{s.a.} \sum_j g_j^{(t)} = d^{(t)} \quad \forall t$$

**BALANCE**

$$g_j^{(t)} \leq u_j^{(t)} G_j^{(\max)} \quad \forall j, t$$

**UNIT COMMITMENT**

$$u_j^{(t)} \leq x_j \quad \forall j, t$$

**INVESTMENT**

$$g_j^{(t)} \in [0, G_j^{(\max)}] \quad \forall j, t$$

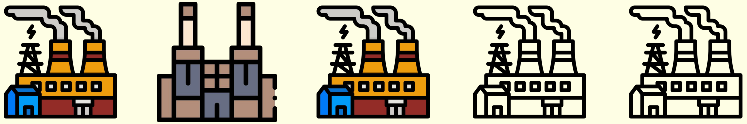
**OPERATION**

$$u_j^{(t)} \in \{0, 1\} \quad \forall j, t$$

**UNIT COMMITMENT**

$$x_j \in \{0, 1\} \quad \forall j$$

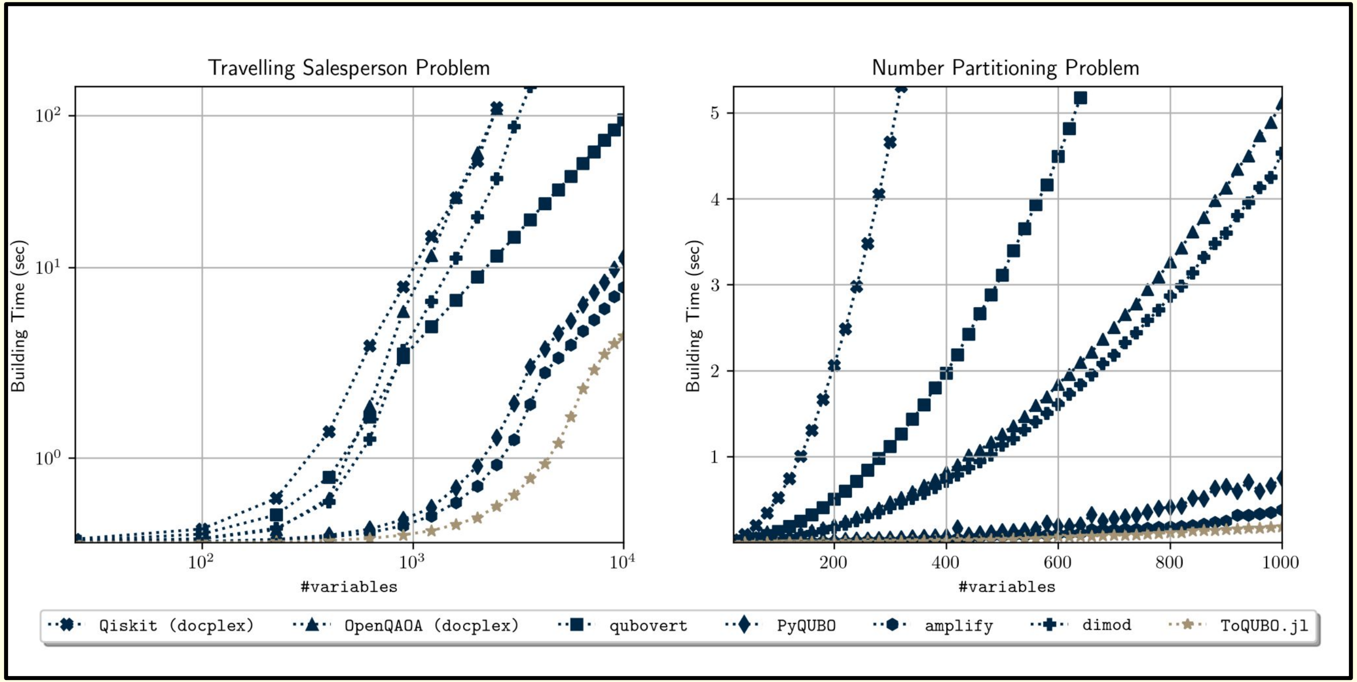
**INVESTMENT**



```julia
using JuMP, QUBO
using PySA

model = Model(() -> ToQUBO.Optimizer(PySA.Optimizer))

@variable(model, 0 ≤ g[1:T,j=1:n] ≤ Gmax[j])
@variable(model, u[1:T,1:n], Bin)
@variable(model, x[1:n], Bin)

@objective(model, Min, sum(c'g[t,:] for t=1:T) + i'x)

@constraint(model, [t=1:T], sum(g[t,:]) == d[t])
@constraint(model, [t=1:T,j=1:n], g[t,j] ≤ u[t,j] * Gmax[j])
@constraint(model, [t=1:T,j=1:n], u[t,j] ≤ x[j])

optimize!(model)

@show objective_value(model)
@show value.(x)
```

# RESULTS

# Reformulation Performance



# Qualitative Analysis

**CITATION ALERT**

TABLE I: Comparing the support provided by of proposed framework and existing libraries and framework in each step of quantum optimization.
✓ indicates that the corresponding action is performed automatically.
✓ signifies that a proper function is available for implementing the step.
✗ indicates that the method is not fully supported.
+ denotes that logarithmic encoding is also compatible with bases different from two.
* signifies that the encoding techniques can be exploited only for constraints translation.
† indicates that the polynomial reduction is implemented by exploiting the corresponding qubovert function.

| Supports for each step | | Existing Libraries | | | | | | Existing Frameworks | | Proposed |
|---|---|---|---|---|---|---|---|---|---|---|
| | | pyqubo [31] | qubovert [32] | dimod [33] | Qiskit [34] | fixstars [35] | openQAOA [36] | AutoQUBO [37] | QUBO.jl [38] | Framework |
| Floating Encoding | | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ | ✓ |
| Integer Encoding | Logarithmic [39] | ✓ | ✓ | ✓ | ✓ | ✓* | ✗ | ✗ | ✓ | ✓+ |
| | Unitary [39] | ✓ | ✓ | ✗ | ✗ | ✓* | ✗ | ✗ | ✓ | ✓ |
| | Dictionary [39] | ✓ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ | ✓ |
| | Domain-Wall [40] | ✓ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ | ✓ |
| | Bounded-Coeff [41] | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ | ✓ |
| | Arithmetic [42] | ✗ | ✗ | ✗ | ✗ | ✓* | ✗ | ✗ | ✓ | ✓ |
| Functions | Equality [22] [21] | ✗ | ✓ | ✓ | ✓ | ✓ | ✓ | ✗ | ✓ | ✓ |
| | Inequality [22] | ✗ | ✓ | ✓ | ✓ | ✓ | ✓ | ✗ | ✓ | ✓ |
| | Boolean [22] | ✓ | ✓ | ✓ | ✗ | ✓ | ✗ | ✗ | ✗ | ✓ |
| Penalty Weight | UB positive [43] | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ |
| | MQC [43] | ✗ | ✗ | ✓ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ |
| | VLM [44] | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ | ✗ | ✓ |
| | MOMC [43] | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ |
| | MOC [43] | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ |
| | UB Naive [45], [46] | ✗ | ✓ | ✗ | ✓ | ✗ | ✗ | ✓ | ✗ | ✓ |
| | UB posiform [45], [46] | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ | ✗ | ✓ |
| Polynomial Reduction | | ✓ | ✓ | ✓ | ✗ | ✓ | ✗ | ✓ | ✓ | ✓† |
| Solvers | Dwave QA | ✓ | ✗ | ✓ | ✗ | ✓ | ✗ | ✓ | ✓ | ✓ |
| | QAOA | ✗ | ✗ | ✗ | ✓ | ✗ | ✓ | ✗ | ✓ | ✓ |
| | VQE | ✗ | ✗ | ✗ | ✓ | ✗ | ✓ | ✗ | ✓ | ✓ |
| | GAS | ✗ | ✗ | ✗ | ✓ | ✗ | ✗ | ✗ | ✗ | ✓ |
| | SA | ✓ | ✓ | ✓ | ✓ | ✓ | ✗ | ✓ | ✓ | ✓ |
| Solution Decoding | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Check Constraints | | ✓ | ✓ | ✓ | ✗ | ✓ | ✗ | ✓ | ✓ | ✓ |
| Penalty Update | Sequential [47] | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ |
| | Scaled [47] | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ |
| | Binary search [47] | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ |

*Towards an Automatic Framework for Solving Optimization Problems with Quantum Computers*, arXiv:2406.12840, 2024

# Qualitative Analysis

**CITATION ALERT**

TABLE I: Comparing the support provided by of proposed framework and existing libra… of quantum optimization.

✓ (green) indicates that the corresponding action is performed automatically.

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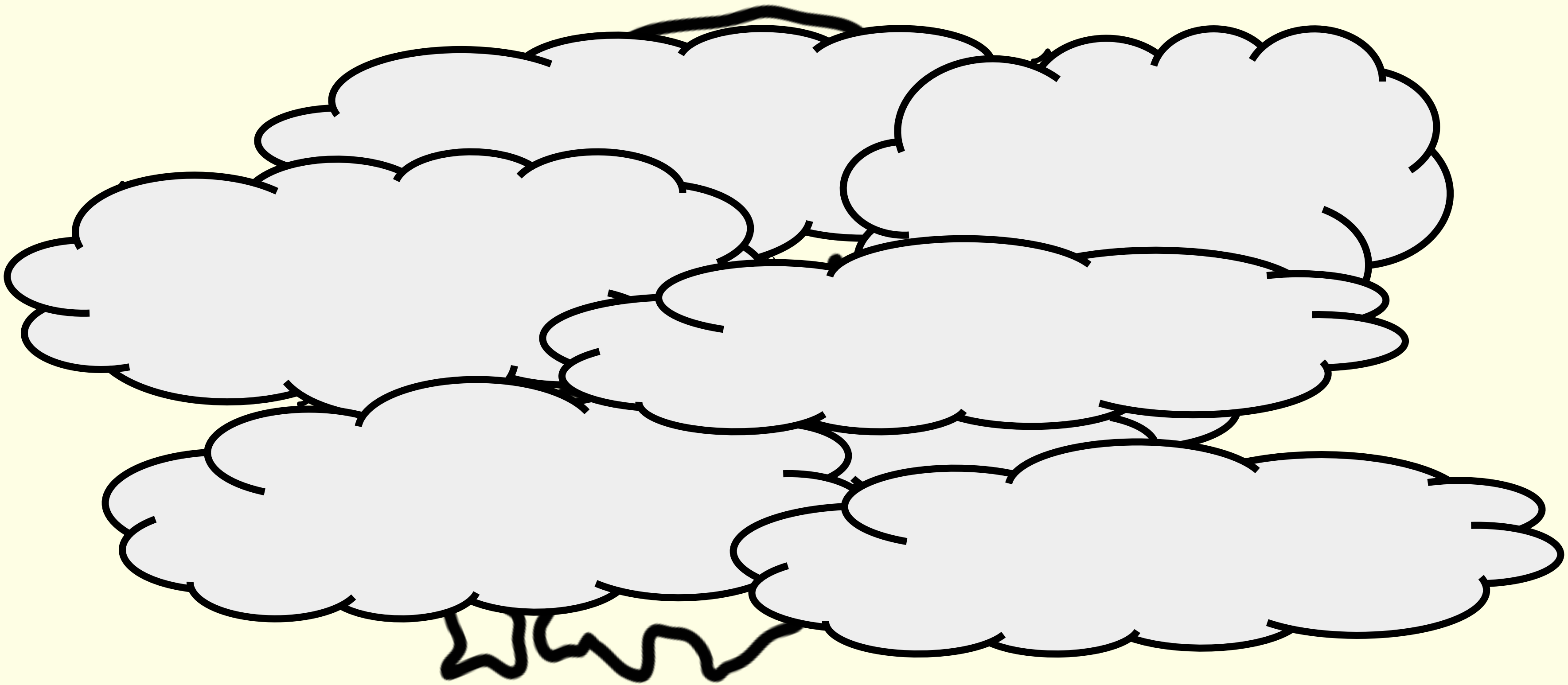
| Floating Encoding | | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ | ✓ |
| Integer Encoding | Logarithmic [39] | ✓ (orange) | ✓ (orange) | ✓ (orange) | ✓ (orange) | ✓* (orange) | ✗ | ✗ | ✓ | ✓+ |
| | Unitary [39] | ✓ (orange) | ✓ (orange) | ✗ | ✗ | ✓* (orange) | ✗ | ✗ | ✓ | ✓ |
| | Dictionary [39] | ✓ (orange) | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ | ✓ |
| | Domain-Wall [40] | ✓ (orange) | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ | ✓ |
| | Bounded-Coeff [41] | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ | ✓ |
| | Arithmetic [42] | ✗ | ✗ | ✗ | ✗ | ✓* (orange) | ✗ | ✗ | ✓ | ✓ |
| … Functions | Equality [22] [21] | ✗ | ✓ (orange) | ✓ (orange) | ✓ (orange) | ✓ (orange) | ✓ (orange) | ✗ | ✓ | ✓ |
| | Inequality [22] | ✗ | ✓ (orange) | ✓ (orange) | ✓ (orange) | ✓ (orange) | ✓ (orange) | ✗ | ✓ | ✓ |
| | Boolean [22] | ✓ (orange) | ✓ (orange) | ✓ (orange) | ✗ | ✓ (orange) | ✗ | ✗ | ✗ | ✓ |
| Penalty Weight | UB positive [43] | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ |
| | MQC [43] | ✗ | ✗ | ✓ (orange) | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ |
| | VLM [44] | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ | ✗ | ✓ |
| | MOMC [43] | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ |
| | MOC [43] | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ |
| | UB Naive [45], [46] | ✗ | ✓ (orange) | ✗ | ✓ | ✗ | ✗ | ✓ | ✗ | ✓ |
| | UB posiform [45], [46] | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ | ✗ | ✓ |
| Polynomial Reduction | | ✓ (orange) | ✓ (orange) | ✓ (orange) | ✗ | ✓ (orange) | ✗ | ✓ | ✓ | ✓† |
| Solvers | Dwave QA | ✓ | ✗ | ✓ | ✗ | ✓ | ✗ | ✓ | ✓ | ✓ |
| | QAOA | ✗ | ✗ | ✗ | ✓ | ✗ | ✓ | ✗ | ✓ | ✓ |
| | VQE | ✗ | ✗ | ✗ | ✓ | ✗ | ✓ | ✗ | ✓ | ✓ |
| | GAS | ✗ | ✗ | ✗ | ✓ | ✗ | ✗ | ✗ | ✗ | ✓ |
| | SA | ✓ | ✓ | ✓ | ✓ | ✓ | ✗ | ✓ | ✓ | ✓ |
| Solution Decoding | | ✓ (orange) | ✓ (orange) | ✓ (orange) | ✓ (orange) | ✓ (orange) | ✓ (orange) | ✓ | ✓ | ✓ |
| Check Constraints | | ✓ (orange) | ✓ (orange) | ✓ (orange) | ✗ | ✓ (orange) | ✗ | ✓ | ✓ | ✓ |
| Penalty Update | Sequential [47] | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ |
| | Scaled [47] | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ |
| | Binary search [47] | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ |

*Towards an Automatic Framework for Solving Optimization Problems with Quantum Computers*, arXiv:2406.12840, 2024

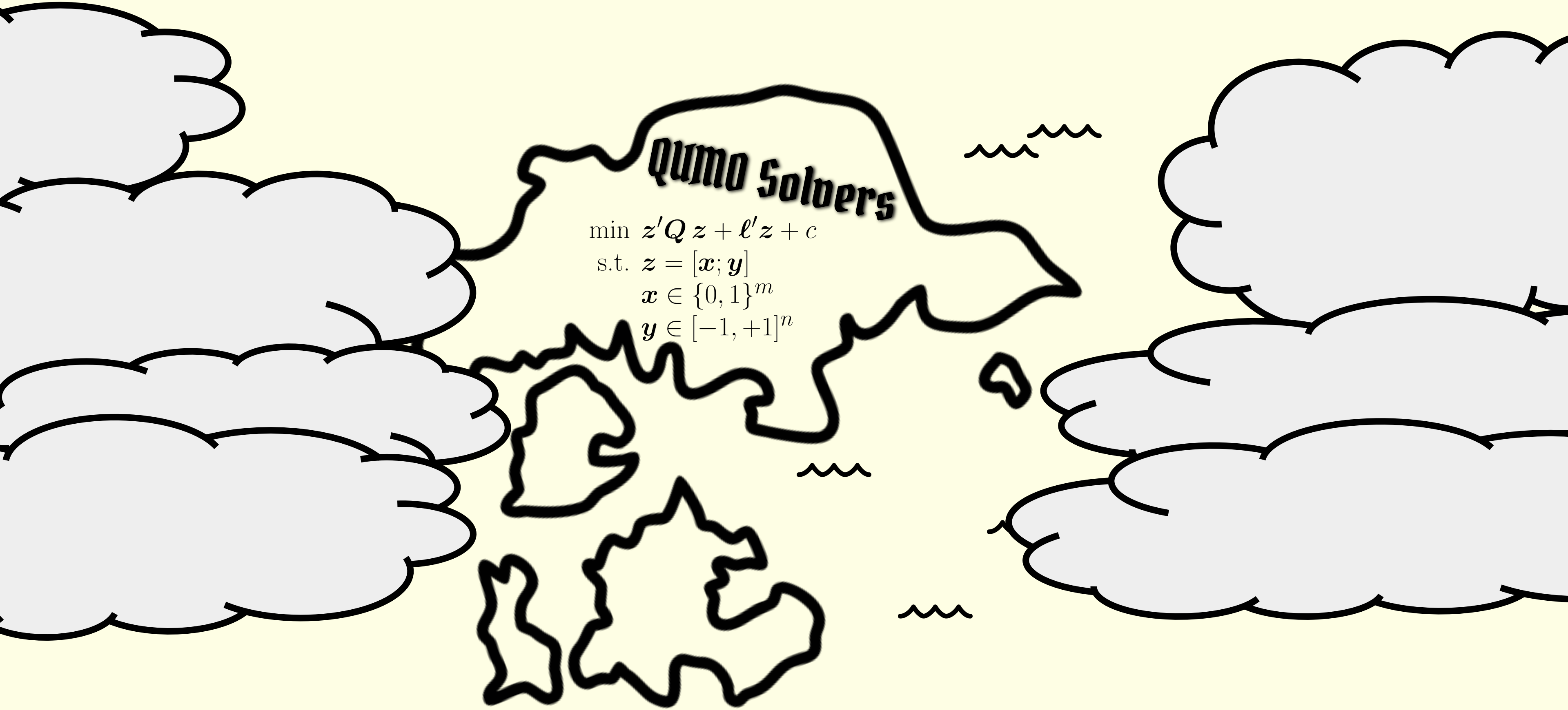
# What's new?

- Additional attributes to control reformulation
  - e.g. `ConstraintPenaltyHint`, `VariableEncodingMethod`
- Reformulation Callbacks
- Architecture-based dispatch
- Disjuctive Programming (`DisjunctiveToQUBO.jl`)



# QUMO Solvers

$$
\begin{aligned}
\min \quad & \boldsymbol{z}'\boldsymbol{Q}\,\boldsymbol{z} + \boldsymbol{\ell}'\boldsymbol{z} + c \\
\text{s.t.} \quad & \boldsymbol{z} = [\boldsymbol{x}; \boldsymbol{y}] \\
& \boldsymbol{x} \in \{0, 1\}^m \\
& \boldsymbol{y} \in [-1, +1]^n
\end{aligned}
$$



# QUMO Solvers

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\text{s.t.} \quad & \boldsymbol{z} = [\boldsymbol{x}; \boldsymbol{y}] \\
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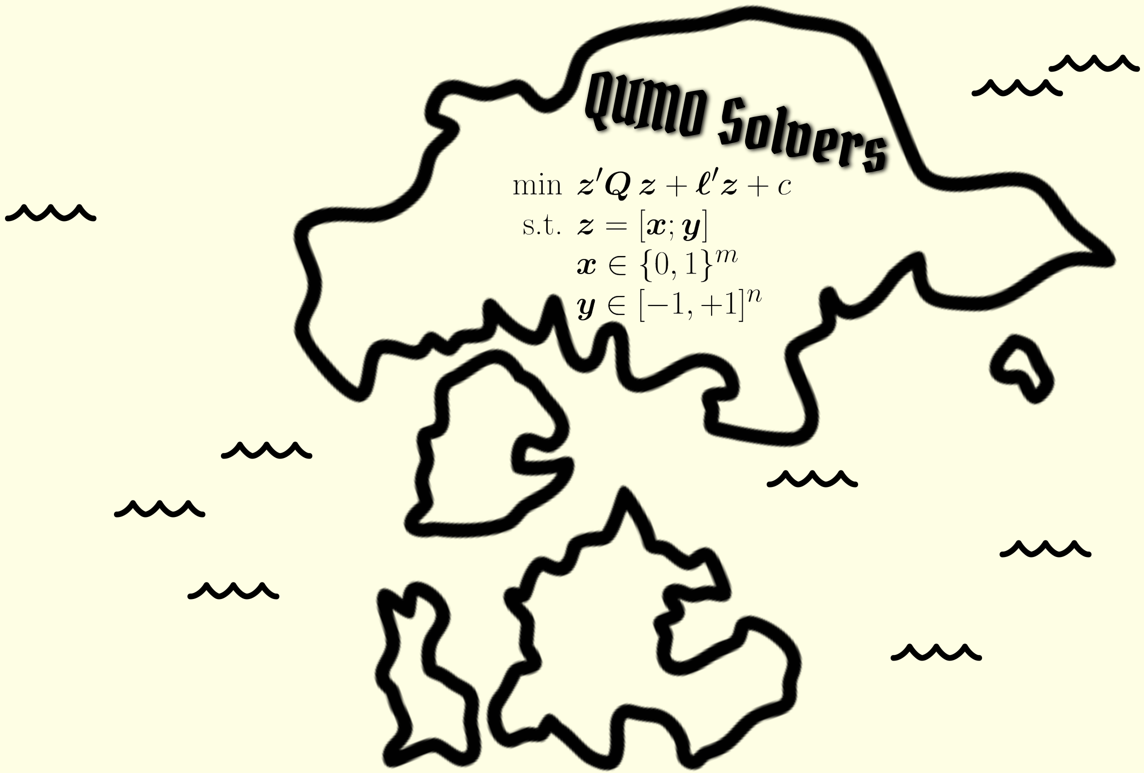
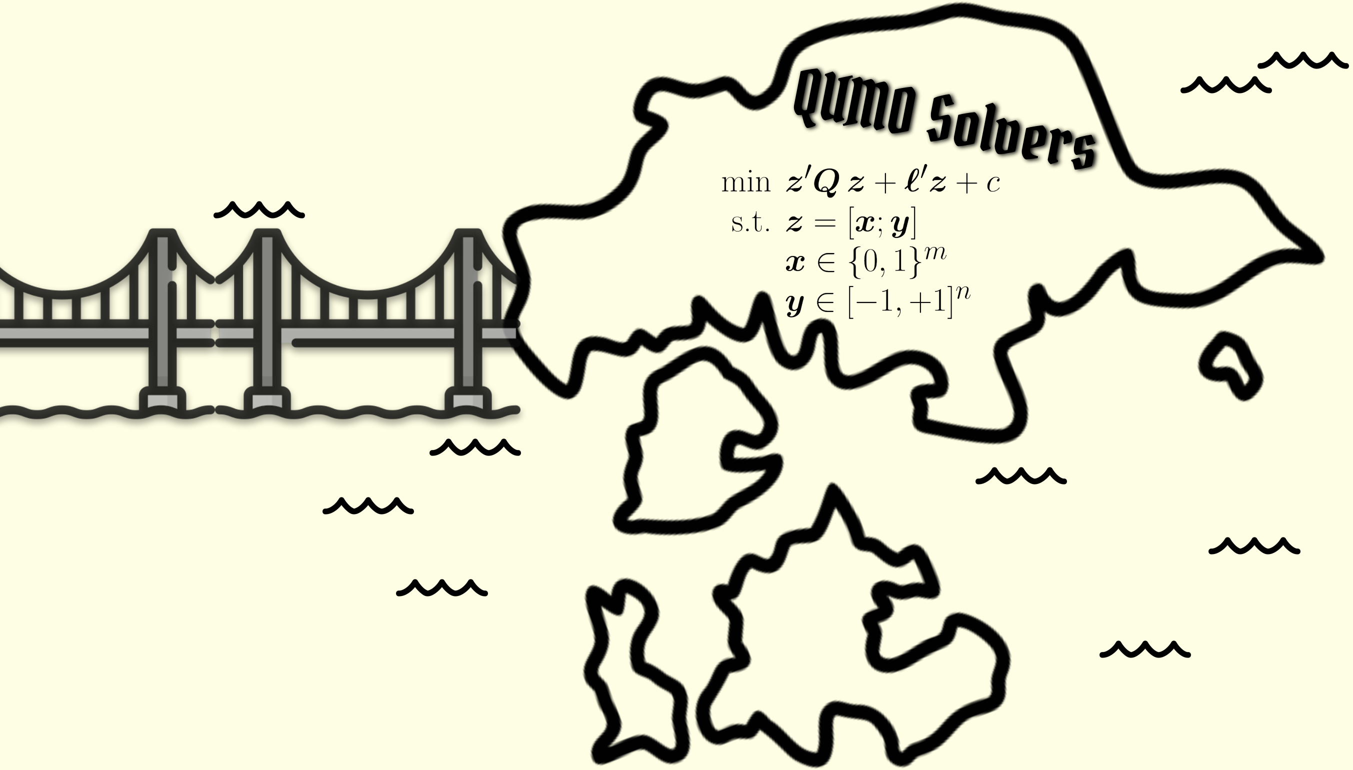
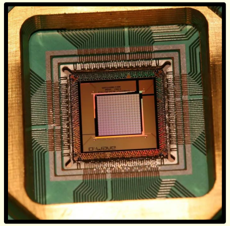


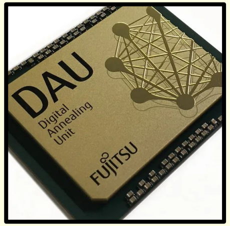



$$
\begin{aligned}
\min\ & \boldsymbol{z}'\boldsymbol{Q}\,\boldsymbol{z} + \boldsymbol{\ell}'\boldsymbol{z} + c \\
\text{s.t.}\ & \boldsymbol{z} = [\boldsymbol{x}; \boldsymbol{y}] \\
& \boldsymbol{x} \in \{0, 1\}^m \\
& \boldsymbol{y} \in [-1, +1]^n
\end{aligned}
$$

# Integrating an heterogeneous Solver Landscape

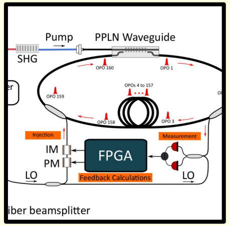


D-Wave

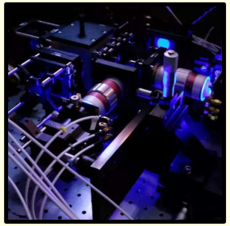



Fujitsu

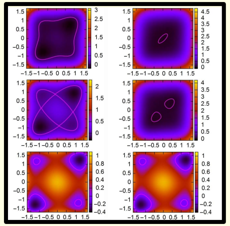
IBM




P. L. McMahon et al., 2016

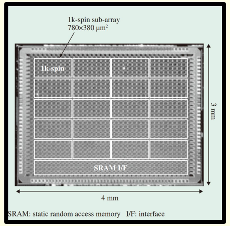


Microsoft Research



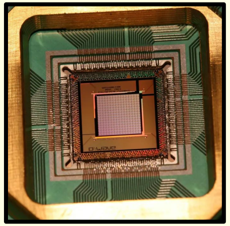
Toshiba, Goto et al., 2019

Julia Mathematical Programming

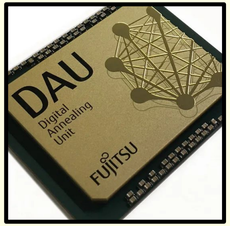



Hitachi, Yamaoka et al.

# Integrating an heterogeneous Solver Landscape

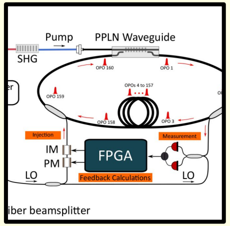


D-Wave

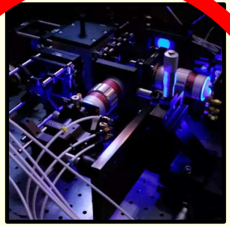



Fujitsu

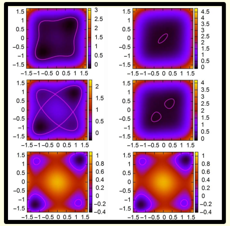
IBM



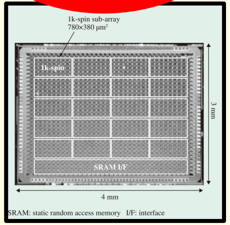

P. L. McMahon et al., 2016



Microsoft Research



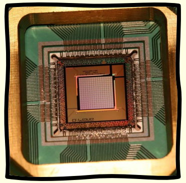
Toshiba, Goto et al., 2019




Hitachi, Yamaoka et al.

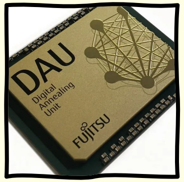
JuMP

Julia Mathematical Programming

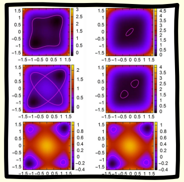


*D-Wave*

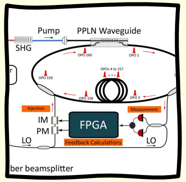
*IBM*



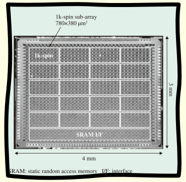

*Fujitsu*



*Toshiba, Goto et al., 2019*

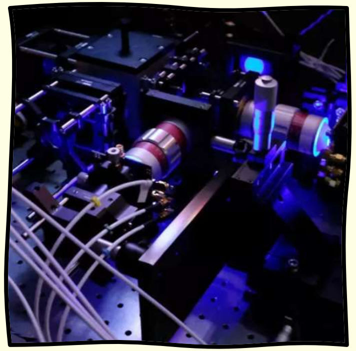



*P. L. McMahon et al., 2016*




*Hitachi, Yamaoka et al.*

$$(\text{QUBO}) \quad \min_{\mathbf{x}} \; \mathbf{x}'\mathbf{Q}\,\mathbf{x} + \boldsymbol{\ell}'\mathbf{x} + c \quad \text{s.t.} \; \mathbf{x} \in \{0,1\}^n$$



***Microsoft Research AOC***

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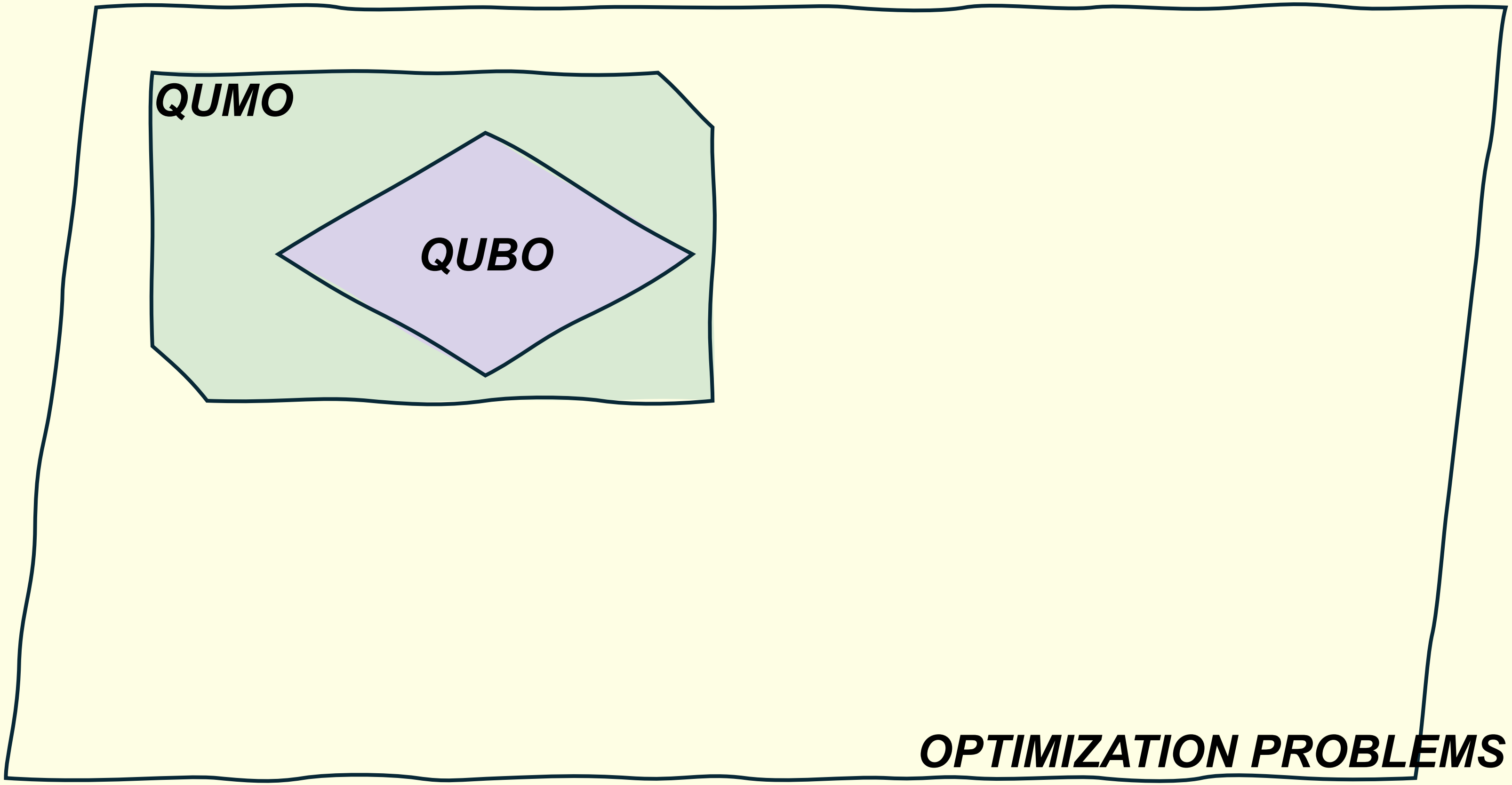

QUMO
QUBO
OPTIMIZATION PROBLEMS

# BREAKING

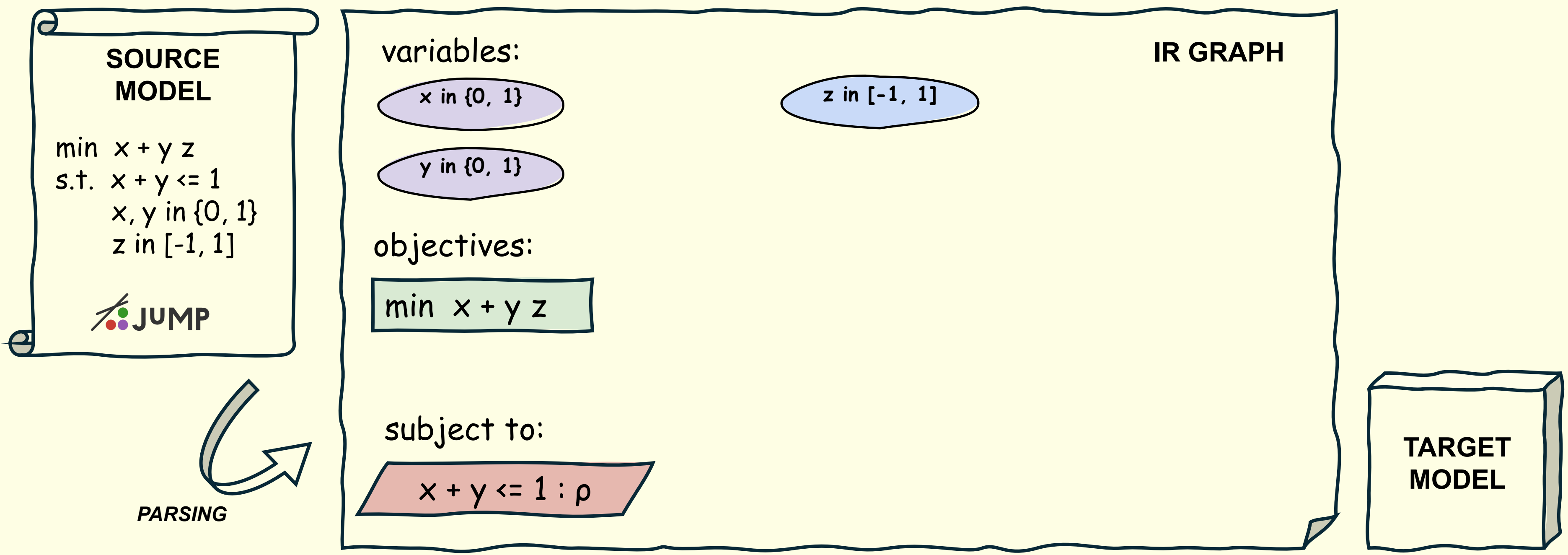
OPTIMIZATION PROBLEMS

**BREAKING**

**PSEUDO-BOOLEAN THEORY IS OVER**

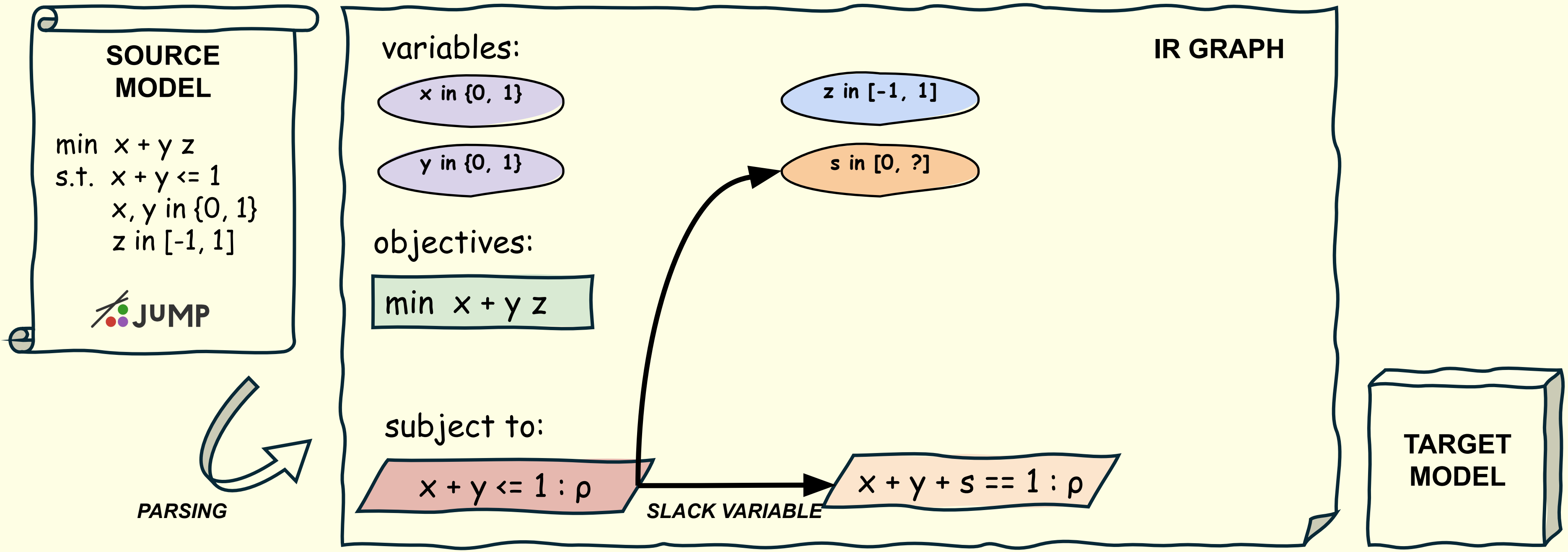
*...TIMIZATI...N PROBLEMS*

# BRIDGING THE GAP




**MOI Bridges:** [2002.03447] MathOptInterface: a data structure for mathematical optimization problems (arxiv.org)

# BRIDGING THE GAP




**MOI Bridges:** [2002.03447] MathOptInterface: a data structure for mathematical optimization problems (arxiv.org)

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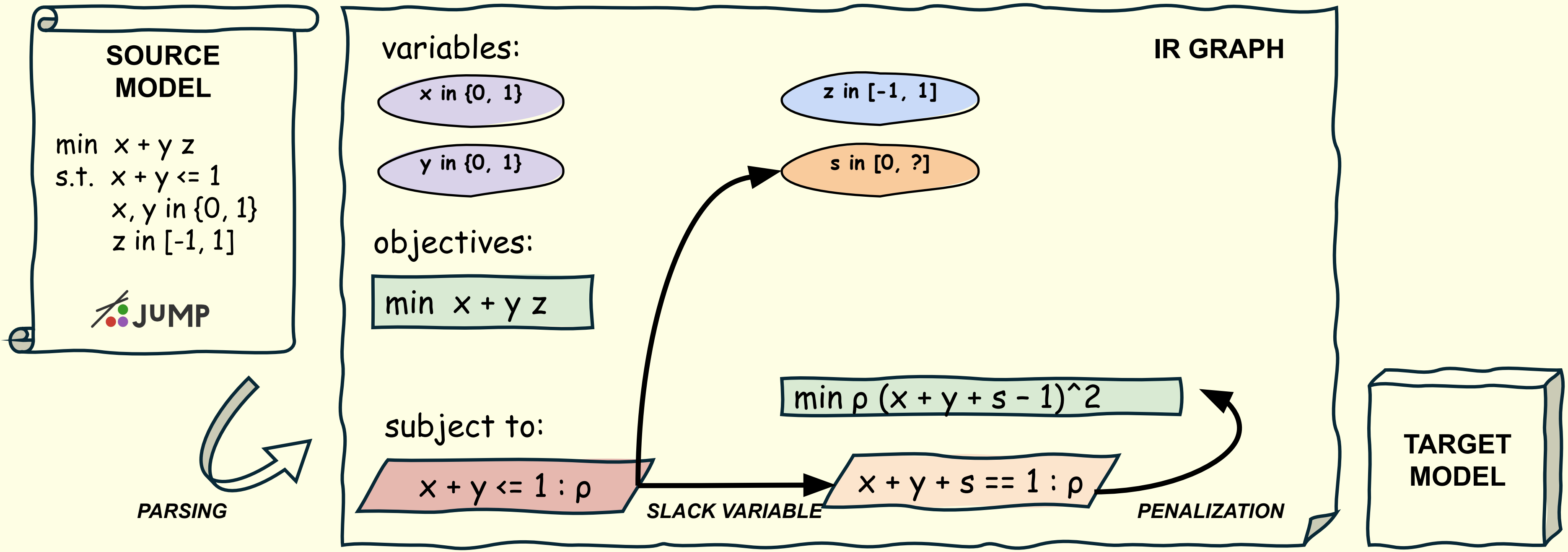


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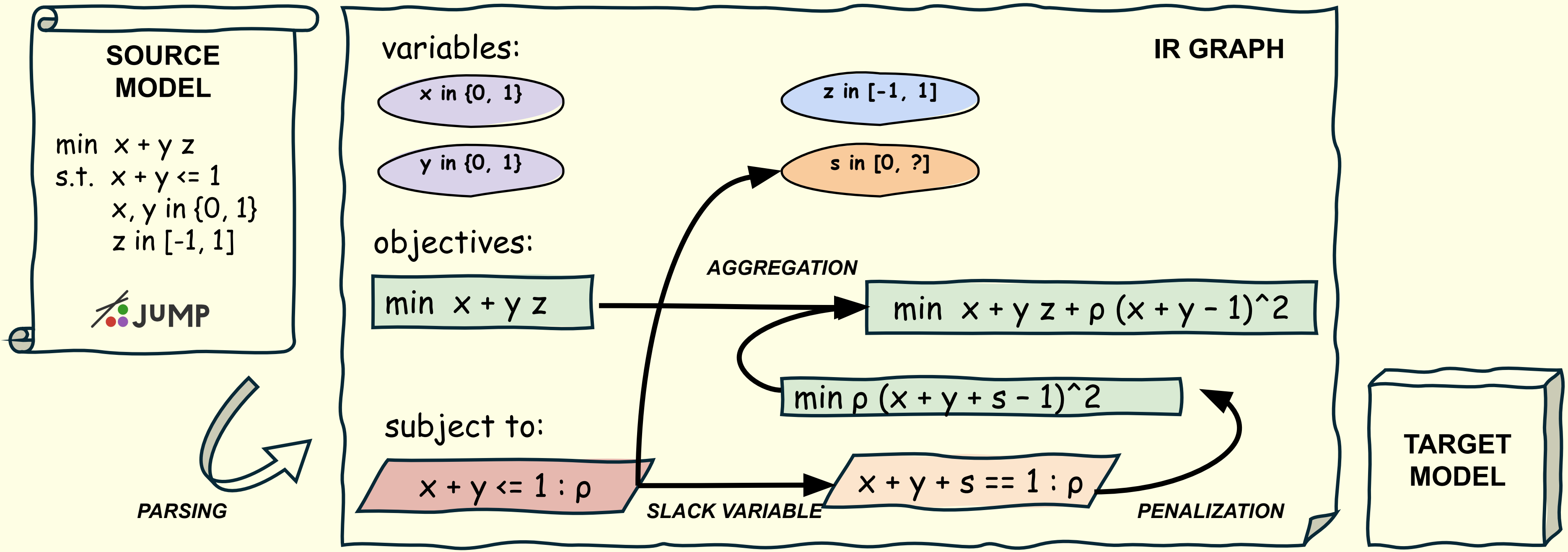


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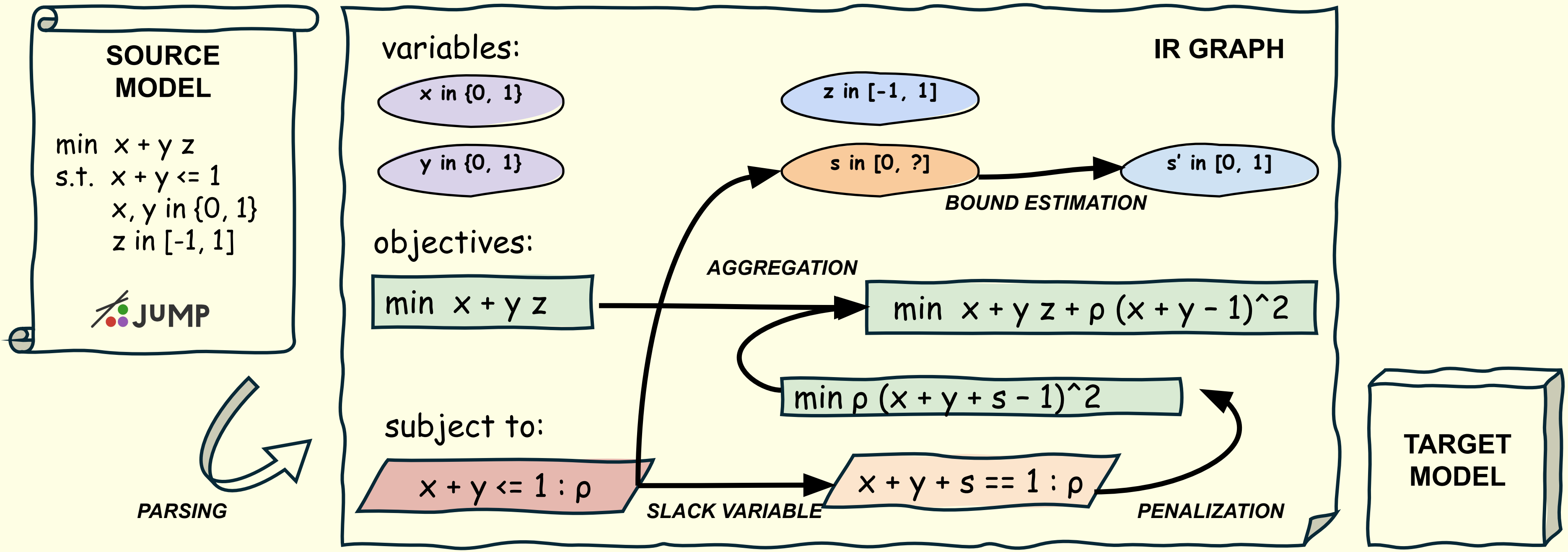


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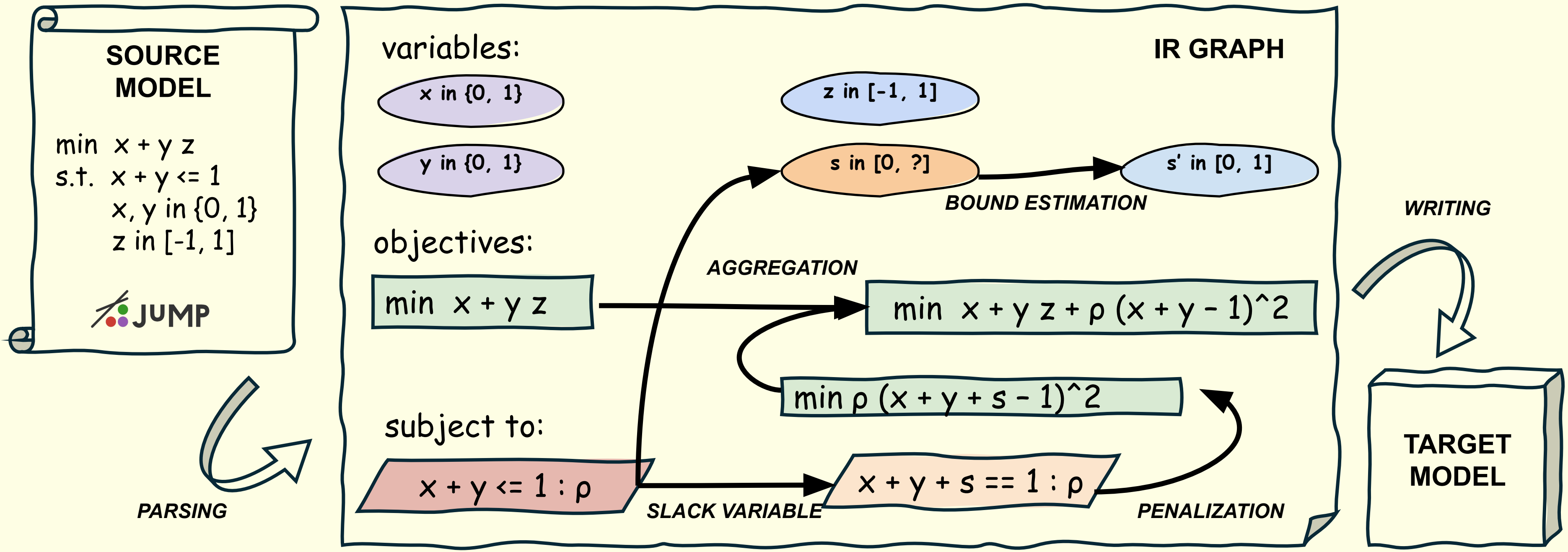



**MOI Bridges:** [2002.03447] MathOptInterface: a data structure for mathematical optimization problems (arxiv.org)

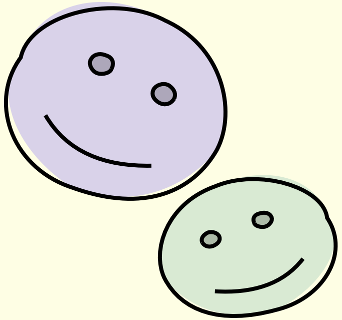
# What about MOI Bridges?

- In fact, it is good to use as many MOI Bridges as possible
- Some of the operations are too destructive
  - e.g. Penalization, Encoding Continuous Variables
- Solver-dependent reformulation path finding
- Fine-grained control over reformulation steps

# Next Steps

- ❏ Generalize the reformulation algorithm
- ❏ Expand constraint support
- ❏ Set up continuous benchmarking service
- ❏ Draw insights from benchmarking to guide reformulation

# ~~Wishlist~~ Food for Thought



- ❑ Asynchronous `optimize!` call
- ❑ Communication protocol for client-server apps

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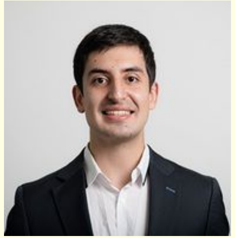

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