THE NEW: DisjunctiveProgramming.jl

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ACKNOWLEDGEMENTS

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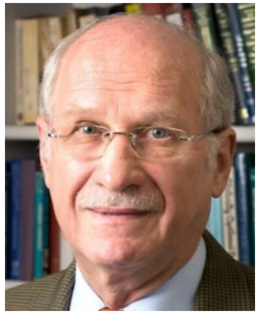

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OUTLINE

- Background
- Modelling API
- Solution Approaches
- Infinite GDP

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- **Background**
- Modelling API
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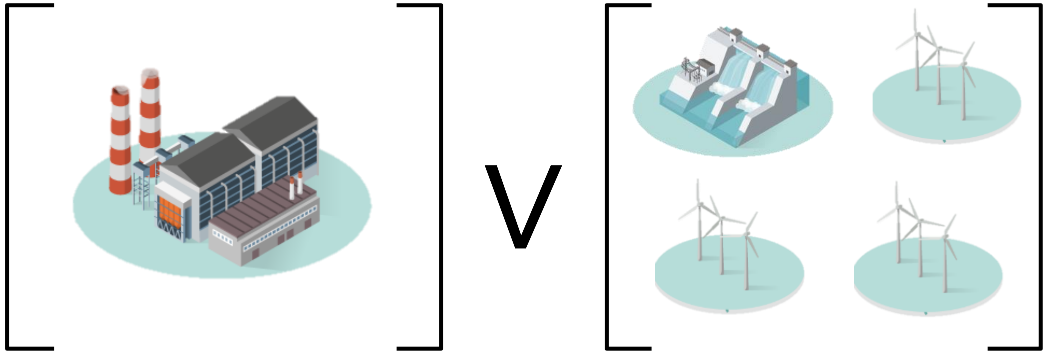


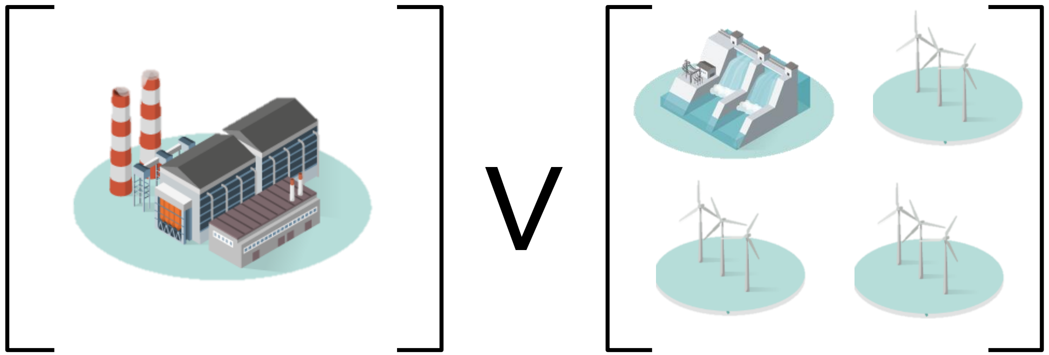
WHAT IS GENERALIZED DISJUNCTIVE PROGRAMMING?

Disjunctions

- **Conditionally enforce constraints** based on values of Boolean variables $G \in [True, False]$

$$\bigvee_{j \in \mathcal{J}_i} \begin{bmatrix} G_{ij} \\ g_{ij}(z) \leq 0 \end{bmatrix}, \quad i \in \mathcal{I}$$

- **Example:** Expansion Planning



Logic Constraints

- Enforce logic on Boolean variables G

$$\Omega(G) = True$$

- Propositional logic

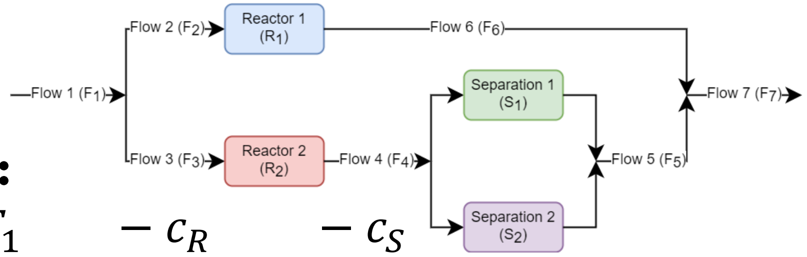
$$\wedge \qquad \vee \qquad \neg \qquad \Longrightarrow \qquad \Longleftrightarrow$$

- Constraint programming logic

$$\text{atleast}(n, \cdot) \qquad \text{exactly}(n, \cdot) \qquad \text{atmost}(n, \cdot)$$

- **Systematic conversion** to algebraic constraints
  - Apply De Morgan's laws to convert to conjunctive normal form

THE OLD



Objective Function:

$$\max \; p_7 \cdot F_7 \quad - p_1 \cdot F_1 \quad - c_R \quad - c_S$$

Global Constraints: *(Material Balances)*

$$F_1 = F_2 + F_3$$
$$F_7 = F_5 + F_6$$

Disjunctions: *(Technology Selection)*

$$\begin{bmatrix} Y_{R_1} \\ F_6 = \beta_{R_1} \cdot F_2 \\ F_3 = F_4 = F_5 = 0 \\ c_R = \gamma_{R_1} \\ c_S = 0 \end{bmatrix} \bigvee \begin{bmatrix} Y_{R_2} \\ F_6 = F_2 = 0 \\ F_4 = \beta_{R_2} \cdot F_3 \\ c_R = \gamma_{R_2} \\ \left[\begin{bmatrix} Y_{S_1} \\ F_5 = \beta_{S_1} \cdot F_4 \\ c_S = \gamma_{S_1} \end{bmatrix} \bigvee \begin{bmatrix} Y_{S_2} \\ F_5 = \beta_{S_2} \cdot F_4 \\ c_S = \gamma_{S_2} \end{bmatrix} \right] \end{bmatrix}$$

Logic Constraints: *(Technology Selection)*

$$Y_{R_1} \veebar Y_{R_2}$$
$$Y_{R_2} \Leftrightarrow \left(Y_{S_1} \veebar Y_{S_2}\right) \quad \leftarrow$$
$$Y_{R_1} \Leftrightarrow \left(\neg Y_{S_1} \wedge \neg Y_{S_2}\right) \quad \leftarrow$$

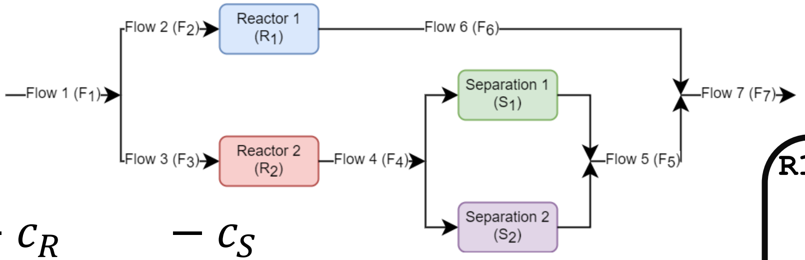
Alternate Propositions

```julia
using JuMP, DisjunctiveProgramming, Random

#parameters
p = Dict(1 => rand(), 7 => 1 + rand())
β = Dict(r => rand() for r in [:R1, :R2])
γ = Dict(r => rand() for r in [:R1, :R2, :S1, :S2])
γSmax = maximum(γ[:S1], γ[:S2])
γRmax = maximum(γ[:R1], γ[:R2])
γRmin = minimum(γ[:R1], γ[:R2])

m = Model()
@variable(m, 0 ≤ F[i = 1:7] ≤ 10)
@variable(m, 0 ≤ CS ≤ γSmax)
@variable(m, γRmin ≤ CR ≤ γRmax)
@constraints(m,
    begin
        F[1] == F[2] + F[3]
        F[7] == F[5] + F[6]
    end
)
inner_disj = @disjunction(m,
    begin
        F[5] == β[:S1]*F[4]
        CS == γ[:S1]
    end,
    begin
        F[5] == β[:S2]*F[4]
        CS == γ[:S2]
    end,
    reformulation = :big_m,
    name = :YS
)
```

← *- Reformulation type*

← *- Indicator Variable*

THE OLD



Objective Function:

$$\max \ p_7 \cdot F_7 - p_1 \cdot F_1 - c_R - c_S$$

Global Constraints: *(Material Balances)*

$$F_1 = F_2 + F_3$$
$$F_7 = F_5 + F_6$$

Disjunctions: *(Technology Selection)*

$$\begin{bmatrix} Y_{R_1} \\ F_6 = \beta_{R_1} \cdot F_2 \\ F_3 = F_4 = F_5 = 0 \\ c_R = \gamma_{R_1} \\ c_S = 0 \end{bmatrix} \bigvee \begin{bmatrix} Y_{R_2} \\ F_6 = F_2 = 0 \\ F_4 = \beta_{R_2} \cdot F_3 \\ c_R = \gamma_{R_2} \\ \begin{bmatrix} Y_{S_1} \\ F_5 = \beta_{S_1} \cdot F_4 \\ c_S = \gamma_{S_1} \end{bmatrix} \bigvee \begin{bmatrix} Y_{S_2} \\ F_5 = \beta_{S_2} \cdot F_4 \\ c_S = \gamma_{S_2} \end{bmatrix} \end{bmatrix}$$

Logic Constraints: *(Technology Selection)*

$$Y_{R_1} \underline{\vee} Y_{R_2}$$
$$Y_{R_2} \Leftrightarrow \left(Y_{S_1} \underline{\vee} Y_{S_2}\right)$$
$$Y_{R_1} \Leftrightarrow \left(\neg Y_{S_1} \wedge \neg Y_{S_2}\right)$$

Alternate Propositions

```julia
R1_con = @constraints(m,
    begin
        F[6] == β[:R1]*F[2]
        F[3] == 0
        F[4] == 0
        F[5] == 0
        CR == γ[:R1]
        CS == 0
    end
)
R2_con = @constraints(m,
    begin
        F[6] == β[:R2]*F[3]
        CR == γ[:R2]
    end
)
add_disjunction!(m,
    R1_con,
    (
        R2_con,
        values(inner_disj)...
    ),
    reformulation = :big_m,
    name = :YR
)
choose!(1, YR[1], YR[2]; mode = :exactly)
choose!(YR[2], YS[1], YS[2]; mode = :exactly)
#@proposition(m, YR[1] ⇔ (¬YS[1] ∧ ¬YS[2]))
```

OLD LIMITATIONS (FROM LAST JUMP-DEV)

- Syntax doesn’t closely match mathematical representation
- Cannot change transformation and resolve
- Doesn’t scale for a large # of disjunctions
- Not compatible with InfiniteOpt.jl
- Doesn’t support nonlinear expressions (due to current JuMP limitations)

OUTLINE

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- **Modelling API**
- Solution Approaches
- Infinite GDP

```
using DisjunctiveProgramming, HiGHS
model = GDPModel(HiGHS.Optimizer)

# Variables
@variable(model, z[j ∈ 1:10])
@variable(model, Y[1:2, 1:10], Logical)
@variable(model, W[1:10], Logical)

# Disjunct 1
@constraint(model, [j ∈ 1:10], 3z[j]^2 <= 4, Disjunct(Y[1, j]))
@constraint(model, [j ∈ 1:10], sin(z[j]) == 42, Disjunct(Y[1, j]))
# Disjunct 2
@constraint(model, [j ∈ 1:10], 2z[j] <= 1, Disjunct(Y[2, j]))
# Disjunction
@disjunction(model, obj_name[j ∈ 1:10], Y[:, j])

# Logic constraints
@constraint(model, [j ∈ 1:10], ¬Y[1, j] ∧ Y[2, j] ⟹ W[j] := true)

# Cardinality constraints
@constraint(model, [j ∈ 1:10], Y[:, j] in AtLeast(1))
```

LOGICAL VARIABLES

- Defined on the set $\{False, True\}$
- Used as indicator for disjuncts and to build logical constraints
- Syntax

```
using DisjunctiveProgramming, HiGHS

model = GDPModel(HiGHS.Optimizer) # wraps a JuMP model

@variable(model, Y[1:2], Logical) # just add the `Logical` tag
```

DISJUNCTIONS

- Disjuncts are identified via an associated logical variable (i.e., the indicator)
- Add constraints to a disjunct via the Disjunct tag
- Create disjunctions w/ @disjunction using the indicator variables

```julia
using DisjunctiveProgramming, HiGHS
model = GDPModel(HiGHS.Optimizer) # wraps a JuMP model
@variable(model, z[j ∈ 1:10])
@variable(model, Y[1:2, 1:10], Logical) # just add the `Logical` tag

# Disjunct 1
@constraint(model, [j ∈ 1:10], 3z[j]^2 <= 4, Disjunct(Y[1, j]))
@constraint(model, [j ∈ 1:10], sin(z[j]) == 42, Disjunct(Y[1, j]))
# Disjunct 2
@constraint(model, [j ∈ 1:10], 2z[j] <= 1, Disjunct(Y[2, j]))
# Disjunction
@disjunction(model, obj_name[j ∈ 1:10], Y[:, j])
```

LOGIC CONSTRAINTS

- Supported logical operators: ∨ (||), ∧ (&&), ¬, ⟹, and ⇔ (==)
- Use JuMP's Boolean constraint syntax using only logical variables
- Also supports cardinality constraints via AtMost, AtLeast, & Exactly sets

```
using DisjunctiveProgramming, HiGHS
model = GDPModel(HiGHS.Optimizer)
@variable(model, Y[1:2, 1:10], Logical)
@variable(model, W[1:10], Logical)

@constraint(model, [j ∈ 1:10], ¬Y[1, j] ∧ Y[2, j] ⟹ W[j] := true)

@constraint(model, [j ∈ 1:10], Y[:, j] in AtLeast(1))
```

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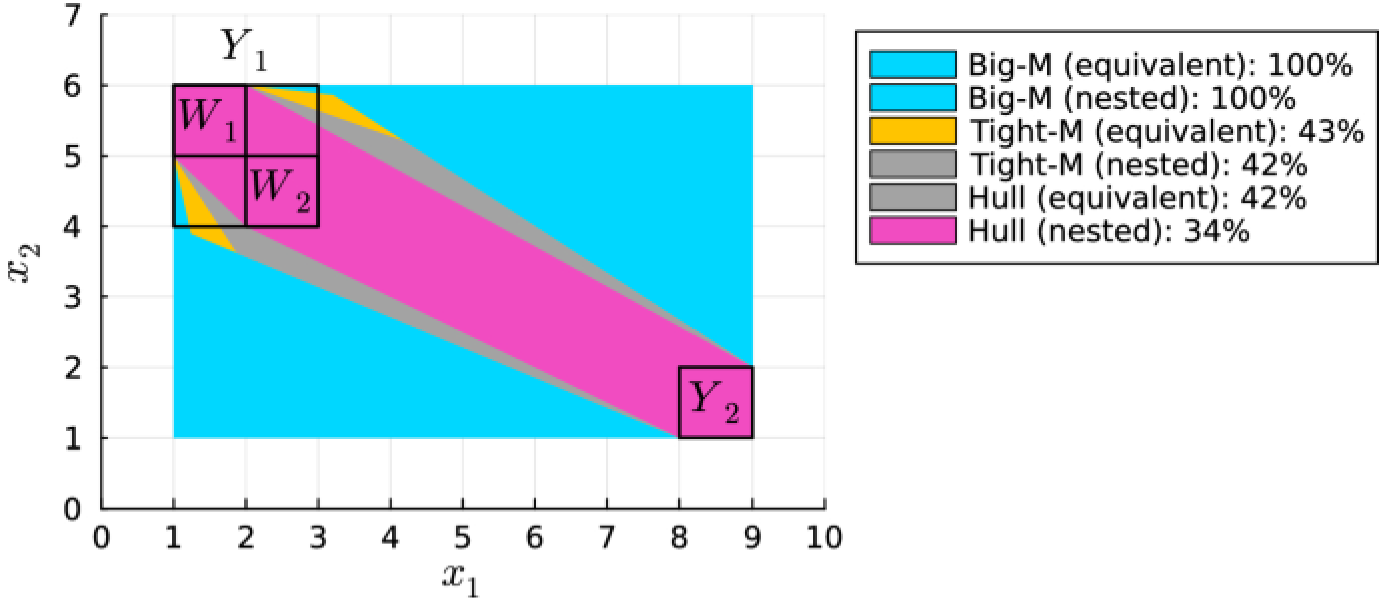

DISJUNCTION REFORMULATIONS

- **Idea:** Convert logic variables to binary & then reformulate disjunctions to MIP
- Reformulates the JuMP model directly (reformulations can be undone too)
- Currently supported reformulations
  - Big-M
  - (Convex) Hull
  - Indicator Constraints

```
optimize!(model, gdp_method = BigM())
optimize!(model, gdp_method = Hull())
optimize!(model, gdp_method = Indicator())
```

- Minimal extension API to add more

LOGIC CONSTRAINT REFORMULATIONS

- Logical constraints are reformulated automatically into (MI)LP constraints
- Accomplishes this by first converting to conjunctive normal form
- Selecting a different approach is not currently supported

```
optimize!(model)
```

FUTURE DEVELOPMENT PLANS

- Create MOI objects and solver to apply more specialized approaches (e.g., LOA)

- Add more reformulation techniques (e.g., multiple big-M, P-split)

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Open [DO NOT MERGE YET] Add InfiniteOpt as an Extension #114

pulsipher wants to merge 4 commits into `master` from `infiniteopt_ext`

```julia
using DisjunctiveProgramming, InfiniteOpt, HiGHS

# Create the model
model = InfiniteGDPModel(HiGHS.Optimizer)

# Create the infinite variables
I = 1:4
@infinite_parameter(model, t ∈ [0, 1], num_supports = 100)
@variable(model, 0 <= g[I] <= 10, Infinite(t))

# Add the disjunctions and their indicator variables
@variable(model, G[I, 1:2], InfiniteLogical(t))
@constraint(model, [i ∈ I, j ∈ 1:2], 0 <= g[i], Disjunct(G[i, 1]))
@constraint(model, [i ∈ I, j ∈ 1:2], g[i] <= 0, Disjunct(G[i, 2]))
@disjunction(model, [i ∈ I], G[i, :])

# Add the logical propositions
@variable(model, W, InfiniteLogical(t))
@constraint(model, G[1, 1] ∨ G[2, 1] ∧ G[3, 1] == W := true)
@constraint(model, 𝔼(binary_variable(W), t) >= 0.95) # incorporate bina

# Reformulate and solve
optimize!(model, gdp_method = Hull())

# check the results
value(W)
```

INFINITE-DIMENSIONAL GDP

- Enforce disjunctions and/or logical constraints with infinite-dimensional variables and/or constraints

$$\bigvee_{j \in \mathcal{J}_i} \begin{bmatrix} G_{ij}(t, x, \xi) \\ g_{ij}(z, y(t, x, \xi)) \leq 0 \end{bmatrix}, \quad i \in \mathcal{I}, t \in \mathcal{D}_t, x \in \mathcal{D}_x, \mathcal{D}_\xi$$

- *Applications:* Strategic planning, expansion planning, scheduling, more

GDP FOR CHANCE CONSTRAINTS

Infinite GDP Derivation

- Chance constraint

$$\int_{\xi \in \mathcal{D}_\xi} \mathbb{1}_{\Omega(g(\xi) \leq 0)}(\xi) p(\xi) d\xi \geq \alpha \qquad (1)$$

- **Disjunctions** over constraint satisfaction

$$\begin{bmatrix} G_i(\xi) \\ g_i(\xi) \leq 0 \end{bmatrix} \underline{\bigvee} \begin{bmatrix} \neg G_i(\xi) \\ g_i(\xi) > 0 \end{bmatrix}, \quad i \in \mathcal{I}, \ \xi \in \mathcal{D}_\xi \qquad (2)$$

- **Express** event logic via logical propositions

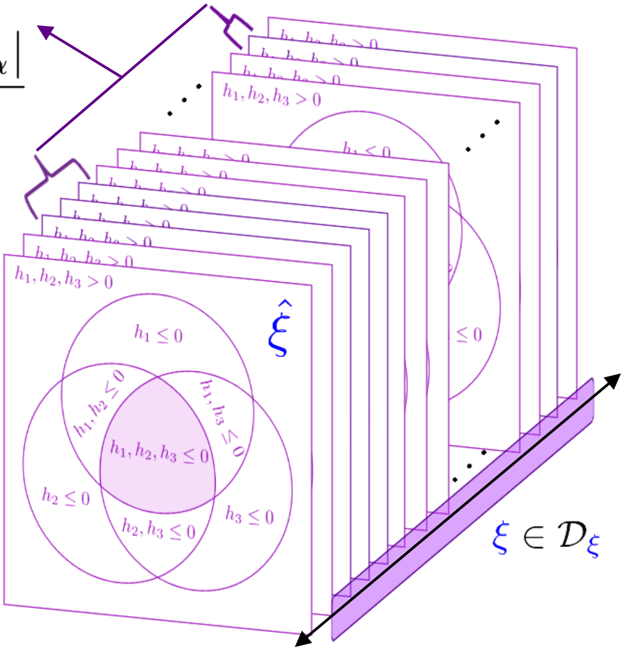
$$\begin{gathered} \Omega(G(\xi)) \iff W(\xi), \quad \xi \in \mathcal{D}_\xi \\ \mathbb{E}_\xi[W(\xi)] \geq \alpha \end{gathered} \qquad (3)$$

- **Combine** (2) and (3) to represent (1)

Illustration

- Event occurs over ***α* fraction** of ξ values*

$$\alpha \leq \frac{1 - |\mathcal{D}_{\xi,\alpha}|}{|\mathcal{D}_\xi|}$$



**Assuming constant pdf*

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J. L. Pulsipher, D. Ovalle, H. Perez, I. E. Grossmann, C. D. Laird. "Characterizing Event Constraints with General Disjunctive Programming". 2022

MODELLING INFINITE GDPS WITH INFINITEOPT

Open **[DO NOT MERGE YET] Add InfiniteOpt as an Extension** #114

pulsipher wants to merge 4 commits into `master` from `infiniteopt_ext`

```julia
using DisjunctiveProgramming, InfiniteOpt, HiGHS

# Create the model
model = InfiniteGDPModel(HiGHS.Optimizer)

# Create the infinite variables
I = 1:4
@infinite_parameter(model, t ∈ [0, 1], num_supports = 100)
@variable(model, 0 <= g[I] <= 10, Infinite(t))

# Add the disjunctions and their indicator variables
@variable(model, G[I, 1:2], InfiniteLogical(t))
@constraint(model, [i ∈ I, j ∈ 1:2], 0 <= g[i], Disjunct(G[i, 1]))
@constraint(model, [i ∈ I, j ∈ 1:2], g[i] <= 0, Disjunct(G[i, 2]))
@disjunction(model, [i ∈ I], G[i, :])

# Add the logical propositions
@variable(model, W, InfiniteLogical(t))
@constraint(model, G[1, 1] ∨ G[2, 1] ∧ G[3, 1] == W := true)
@constraint(model, 𝔼(binary_variable(W), t) >= 0.95) # incorporate bina

# Reformulate and solve
optimize!(model, gdp_method = Hull())

# check the results
value(W)
```

- Extension that loads w/ InfiniteOpt
- Adds InfiniteGDPModel and InfiniteLogical
- More efficient reformulations

DisjunctiveProgramming.jl

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